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Market Clearing Mechanisms under Uncertainty

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Electricity markets face a substantial amount of uncertainty. While traditionally this uncertainty has been due to varying demand, with the integration of larger proportions of volatile renewable energy, added uncertainty from generation must also be faced. Conventional electricity market designs cope with uncertainty by running two markets: a market that is cleared ahead of time, followed by a real-time balancing market to reconcile actual realizations of demand and available generation. In such mechanisms, the initial clearing process does not take into account the distribution of outcomes in the balancing market. Recently an alternative so-called stochastic settlement market has been proposed (see e.g. Pritchard et al. (2010) and Bouffard et al. (2005)). In such a market, the ISO clears both stages in one single settlement market.

In this paper we consider simplified models for two types of market clearing mechanisms. First a market clearing mechanism utilized in New Zealand, whereby firms offer in advance and are notified of a clearing quantity and price guide based on an estimate of demand. Then in real time firms are dispatched according to realized demand and prices are determined. We refer to this as NZTS. Secondly we consider a simplified stochastic programming market clearing mechanism.

We demonstrate that under the assumption of symmetry, our simplified stochastic programming market clearing is equivalent to a two period single settlement (TS) system that takes count of deviation penalties in the second stage. These however differ from NZTS. We prove that this stochastic settlement models results in better social welfare than the NZTS under the assumption of symmetry.

Our models are targeted towards analyzing imperfectly competitive markets. We will construct Nash equilibria of the resulting games for the introduced market clearing mechanisms and compare them under the assumptions of symmetry and in an asymmetric example.

Key words: Uncertainty, Stochastic Programming, Optimization, Equilibrium

1. Introduction

Electricity markets face two key features that set them apart from other markets. The first is that electricity cannot be stored, so demand must equal supply at all times. This is particularly problematic given that demand for electricity is usually uncertain. Second, electricity is transported from suppliers to load over a transmission network with possible constraints. The combination of these two features means that in almost all electricity markets today an Independent System Operator (ISO) sets dispatch centrally and clears the market. Generators and demand-side users can make offers and bids, and the ISO will choose which are accepted according to a pre-determined settlement system.

The classic settlement system used in almost all existing electricity markets is one where the ISO sets dispatch to maximize social welfare. Effectively the ISO matches supply to meet (the uncertain) demand at every moment while maximizing welfare. This becomes particularly difficult in the short-run (up to 24 hours before actual market clearing) as some types of generator (e.g. steam turbines and to some extent gas turbines) need to ramp up their generation slowly, and it is costly to change their output rapidly. Different markets have approached this problem in different ways.

One approach used is to run a deterministic two-period market clearing model (see e.g. Kamat and Oren (2004)). In many jurisdictions such as the PJM, this amounts to a day ahead followed by a real time market with separate, financially binding settlements for each market. In New Zealand, however, the generators can place offers for a given half hour period up to "gate closure" which occurs two hours prior to a designated period. At gate closure these offers are locked in. Estimates of dispatch quantity and price are provided to the industry participants based on forecast demand in the periods leading up to real time. We refer to these as pre-dispatch quantities and prices.[1] At the start of the designated period an accurate measure of demand is available to the ISO and the generators are dispatched accordingly. In the New Zealand electricity market (NZEM), the ISO redispatches the generators every five minutes during a half hour period, using updated demand information but according to the same offer curves, locked in at gate closure.[2] In the NZEM there is only a single settlement as the pre-dispatch quantities and prices are not financially binding. In this deterministic two period single-settlement (NZTS) market, expected demand is used to clear the pre-dispatch quantities and the ISO has no explicit measure of any deviation costs for a generator.

[1] In so far as using these estimates to make an offer, the information is only useful up to gate closure as thereafter offers can not be changed.

[2] The current financial settlement in the NZEM is based on ex-post prices that are computed with average demand over a period. Constrained on and off payments are used to ensure sufficient payment is made to the generators. However the Electricity Authority has taken real time pricing under consultation with the stakeholders.

An alternative to deterministic settlement systems is to use a stochastic settlement process to deal with variable demand. (We will consider demand as gross demand net intermittent renewable generation such as wind, which in the NZEM is forced to offer at zero price.) In a stochastic settlement, the ISO can choose both pre-dispatch and short-run deviations for each generator to maximize expected social welfare in one step. We might then expect a stochastic settlement system to do better (on average) than a deterministic two period system. The idea of a stochastic settlement can be attributed to Bouffard et al. (2005), Wong and Fuller (2007) and Pritchard et al. (2010). In these two-stage, single settlement models, the pre-dispatch clears with information about the future distribution of uncertainties in the system (e.g. demand and volatile renewable generation,) and information about deviation costs for each generator. These models assume that each firms' offers and deviation costs are truthful. In an imperfectly competitive market, this assumption is not valid. The question then remains: can the stochastic settlement auction give better expected social welfare when firms are behaving strategically? That is the question explored by this paper.

In order to answer this question, we should first construct an equilibrium model of these two market mechanisms.

There are several studies that analyse supply function equilibrium (SFE) models in the literature. Klemperer and Meyer (1989b) were the first authors who considered uncertainty in demand for SFE models. In their model, firms are uncertain about demand and should cater for different demand scenarios, as the spot price is determined after demand realization and is dependent on their offered supply function. They state that having uncertainty in the model decreases the number of equilibria dramatically as generators need to come up with supply functions that are optimal for different cases of residual demand functions.

There are also several studies that consider deterministic two settlement electricity markets. Allaz and Vila (1993), Willems (2005) and Gans et al. (1998) show that firms are desirous of participating in a forward market, even though it decreases prices. Allaz (1992) and Allaz and Vila (1993) consider a two settlement Cournot-Nash electricity market for the first time. Their model illustrates how firms are exposed to Prisoner's dilemma leading them to contract in a forward market and ultimately reduce spot prices and their respective profits. Haskel and Powell (1994) extended Allaz and Vila's analysis for general conjectural variations. Gans et al. (1998) also upheld the conclusion that a contract market can increase competition in the spot market and so leads to lower prices. They used a two-way contract for their model. Willems (2005) replaced two-way contracts in Allaz and Vila's model with call options and compared the consequences of these assumptions, such as market efficiency. Later, Bushnell (2007) extended Allaz and Vila's model to an oligopoly and investigated the effects of forward contracting on the spot market. Su (2007) also extended Allaz and Vila's model to a case of non-symmetric firms and concluded that forward market increases market efficiency. While these models are interesting, they fall short of analysing a situation like what occurs in New Zealand. Our focus for NZTS is to address the effect of costs due to dispatch deviations, resulting from short term (a few trading periods) variations in net demand, to generator offers.

There are also a number of models in the literature that consider uncertainty in the context of two settlements.

von der Fehr and Harbord (1992) used a two settlement model with multiple demand scenarios and capacity constraints. They explained how spot prices could be reduced as an effect of contract forward markets. They argued that willingness to increase contract quantity and to be dispatched up to their full capacity in most scenarios causes firms to take part in the contract market. Also on this matter, Batstone (2000) examined the problem of uncertainty in cost of generation along with risk-neutral generators and risk-averse consumers and found that generators are willing to take advantage of the uncertainty of consumers towards the spot market. The strategic behaviour of generators increases the uncertainty and risk of the spot market for consumers.

Shanbhag (2006) investigates a two settlement stochastic market for a two-node model. The model he considers in his investigation is a Nash-Cournot model. Zhang et al. (2010) propose a two stage oligopoly stochastic Nash-Cournot equilibrium problem with equilibrium constraints. This means that their forward market equilibrium is constrained to the spot market equilibrium as a complementarity problem. In their model, they use rational risk-neutral generators that are involved in a two-way contract market. The spot market is assumed to be a Nash-Cournot market. They demonstrate the existence and uniqueness of equilibrium, and use some numerical examples to show these characteristics in a market implementation. None of these models caters for the recently proposed stochastic programming market clearing model that we investigate in this paper.

We start by introducing a simplified version of the NZTS market currently operated in New Zealand. We will then introduce a simplified version of the stochastic programming mechanism for clearing electricity markets. The first simplification in our models is to use affine supply function offers. Green (see Green (1996)) used this restricted form of supply functions to assess the effects of policies on enhancing competition in the England and Wales market. He showed that in presence of linear demand functions, one supply function equilibrium is always an equilibrium over affine supply functions. Subsequently, Baldick et al. (2004) extended Green's work to piecewise linear supply functions. Day et al have also used linear supply functions but with specific forms of conjectural variation (see Day et al. (2002)).

We will establish that the stochastic program reduces to a two period single settlement model slightly different from the NZTS model were deviation penalties are explicitly considered by the

ISO. We refer to this market clearing mechanism as ISOSP. We will present existence results of equilibria for the simplified NZTS and derive an analytical expression for a symmetric equilibrium. We then establish the key result that reduces the simplified stochastic market clearing mechanism (ISOSP,) to a NZTS type model, but with explicit deviation penalties. Here again we construct analytical expressions for symmetric equilibria. Finally we compare the symmetric equilibria of NZTS and ISOSP settlements and show that the ISOSP settlement with explicit deviation costs performs better in terms of expected social welfare. When the assumption of symmetry is relaxed, analytical expressions for an equilibrium become intractable for either model. We will therefore present numerical results for a number of cases and compare equilibria of the two market clearing systems. We find that in all cases ISOSP performs better than the NZTS.

In section 6.2, we repeat the symmetric experiments that construct and compare the equilibria of NZTS and ISOSP mechanisms, but for the restricted version of linear supply functions where the intercept is set to zero. Here we find that there is a unique symmetric equilibrium for the game. For this case, we present an example where the ISOSP welfare is in fact less than the NZTS mechanism. This result is in contrast to the previous set of results where generators have another degree of freedom in their bids in the form of an intercept. Section 7 concludes the paper.

2. The Market Environment

In this paper, we aim to compare different market designs for electricity. We begin by presenting assumptions that are common to all markets we consider, features of what we call the *market environment*. These include such considerations as the number of firms, the costs firms face, the structure of demand and so forth.

Assumption 1. *The market environment may be defined by the following features.*

- *Electricity is traded over a network with no transmission constraints and no line losses, thus we may consider all trading as taking place at a single node.* [3]
- *Demand for electricity is uncertain, and may realize in one of $s \in \{1, \cdots, S\}$ possible outcomes (scenarios), each with probability θ_s. Demand in state s is assumed to be linear, and defined by the inverse demand function $p_s = Y_s - ZC_s$, where C_s is the quantity of electricity and p_s is the market price, in scenario s. Without loss of generality, assume $Y_1 < Y_2 < ... < Y_S$. We will denote the expected value of Y_s by $Y = \sum_s \theta_s Y_s$. The distribution of demand is common knowledge to the agents.*
- *There are n symmetric firms wishing to sell electricity.*

[3] This assumption may also shed light on any scenario where line capacities do not bind, even if in other scenarios they do bind.

- *For a given firm i in scenario s, we will denote the pre-dispatch quantity by q_i, and any short-run* change *in production by $x_{i,s}$. Thus a generator's actual production in scenario s is equal to $q_i + x_{i,s}$, which we denote by $y_{i,s}$.*
- *Each firm i's* long-run *cost function is $\alpha q_i + \frac{\beta}{2} q_i^2$, where q_i is the quantity produced by firm i, and $\beta > 0$.*
- *Each firm's* short-run *cost function is $\alpha (q_i + x_{i,s}) + \frac{\beta}{2} (q_i + x_{i,s})^2 + \frac{\delta}{2} x_{i,s}^2$, where q_i is the expected dispatch of firm i, and $q_i + x_{i,s}$ is the actual short-run dispatch and $\delta > 0$.*
- *As minimum marginal cost of generation should not be more than maximum price of electricity, we assume*

$$\alpha \leq Y_s \qquad \forall s \in \{1, \dots, S\}.$$

- *There is an Independent System Operator (ISO) who takes bids and determines dispatch and prices according to the given market design.*
- *All the above assumptions are common knowledge to all participants in the market.*
- *We assume that the strategy space of each participant is defined by their choice of linear supply function parameters discussed under each market clearing mechanism.*
- *The payoff for each participant is determined by revenues earned from the spot sales of electricity less the cost of production defined above.*

Our assumptions on generators' cost functions are particularly critical to the analysis that follows, and deserve further explanation. Generators face two distinct costs when generating electricity. If given sufficient advance notice of the quantity they are to dispatch, the generator can plan the allocation of turbines to produce that quantity most efficiently. This is what we mean by a *long-run* cost function. The interpretation of this is the lowest possible cost at which a generator can produce quantity q. In electricity markets, however, demand fluctuates at short notice, and the ISO may ask a generator to change its dispatch at short notice. In this case, generators may not have enough time to efficiently reallocate its turbines. For example, many thermal turbines take hours to ramp-up. Most likely, the generator will have to adopt a less efficient production method, such as running some turbines above their rated capacity which also increases the wear on the turbines. Thus there is some inherent cost in deviating from an expected pre-dispatch in the short-run. This cost can be incurred even if the requested deviation is negative. We assume that the generator will be unable to revert to the most efficient mode of producing this quantity $q_{i,s} + x_{i,s}$ in the short-run, so pays a penalty cost. Note that this imposes a positive penalty cost upon the generator for making the short-run change, even if the change is negative. This penalty cost is additively imposed on top of the 'efficient' cost of producing at the new level. We call this cost the *deviation cost*. Note that we assume the symmetric case in which cost of generation and deviation is determined through the same constant parameters (α, β, δ).

Our goal is to compare the outcomes of different markets imposed upon this environment. To be able to draw comparisons in different paradigms, we need to examine the steady state behaviour of participants under the different market clearing mechanisms. To this end, we need to compute equilibria arising under the different market clearing mechanisms. In order to make the computations tractable, we will restrict the firms to offer linear supply functions in the following sections of this paper.

3. Deterministic Two Period Settlement (NZTS) Model

In this section we will introduce a deterministic two period market which is inspired by the market clearing mechanism as it operates currently in New Zealand. As explained in the introduction, in the NZEM firms bid a step supply function for a given half hour period. The bid is made at least two hours in advance. Once gate closure occurs (two hours in advance of any given period), the supply function offers can not be changed. The market will then be cleared six times, every five minutes during the given half hour period. Each five minute re-dispatch is computed with real time demand, but with the supply offer stacks that have been submitted prior to gate closure.[4] We simplify the situation by assuming the market clears only twice; once after the offers are submitted, but before demand is realized. This we call the 'pre-dispatch' phase which tells the generators approximately how much they should produce. Once demand is realized, the same offers will be used to determine actual dispatch in what we call the 'spot settlement'. The difference between pre-dispatch and spot dispatch is a generator's short-run deviation, which is subject to potentially higher costs as we described earlier however the ISO has no knowledge of this cost and it is not explicitly stated in the generators' bids. This cost can be indirectly reflected in the supply functions the generators bid in.

3.1. Mathematical Model

Our simplified mathematical model for the NZTS market has two distinct stages; pre-dispatch and spot. Following the large body of literature on affine supply functions (see e.g. Green (1996), Baldick et al. (2004)) we will work with a linear demand curve and frame generator supply offers as linear functions. Explicitly, each generator i bids a supply function $a_i + b_i q_i$ before the pre-dispatch market to represent their quadratic costs. This supply function is required to be increasing, i.e. the

[4] Within this five minute period a frequency keeping generator will match any small changes in demand. We ignore this aspect of the market, as frequency-keeping is purchased through a separate market and until recently was procured through annual contracts.

offered b_i must satisfy $b_i \geq \varepsilon > 0$, where ε is the machine epsilon. Note that unlike Green (1996), we do not assume that the intercept $a_i = 0$. Following our main analysis, we will present a brief special case where $a_i = 0$ is required.

When generators lock in their offers demand is uncertain (in New Zealand this point in time is referred to as *gate closure*). The ISO will then use the generator's bid twice: once to clear the pre-dispatch market, and once again after demand is realized to clear the spot market. The pre-dispatch market determines the pre-dispatch quantities each generator is asked to dispatch, and the spot market determines the final quantities the generators are asked to dispatch. As in reality, in both the pre-dispatch and spot markets, the ISO aims to maximize social welfare, assuming generators are bidding their true cost functions. Since demand is unknown in pre-dispatch, the ISO will nominate (and use) an expected demand (and will not consider the distribution of demand).

$$\begin{array}{ll} \min z = \sum_{i=1}^{n} \left(a_i q_i + \frac{b_i}{2} q_i^2\right) - \left(YQ - \frac{Z}{2} Q^2\right) & \\ s.t. \quad \sum_{i=1}^{n} q_i - Q = 0 & [f] \end{array} \tag{1}$$

From this first settlement, the ISO can extract a forward price f equal to the shadow price on the (expected demand balance) constraint. f is not used for any settlements as the pre-dispatch quntity and prices are merely a guide at this stage. Recall that the pre-dispatch quantity for generator i is denoted by q_i. After pre-dispatch is determined, true demand is realized, and the ISO then clears the spot market (using the specific demand scenario that has been realized) to maximize welfare by solving (2).

$$\begin{array}{ll} \min z = \sum_{i=1}^{n} \left(a_i y_{i,s} + \frac{b_i}{2} y_{i,s}^2\right) - \left(Y_s C_s - \frac{Z}{2} C_s^2\right) & \\ s.t. \quad \sum_{i=1}^{n} y_{i,s} - C_s = 0 & [p_s] \end{array} \tag{2}$$

Here again the ISO computes a spot price p_s as the shadow price on the constraint. (Note that we can eliminate the constraint and substitute C_s in the objective, however imposing this constraint enables the easy introduction of the price as the shadow price attached to the constraint.) The generator is not permitted to change its bid after pre-dispatch, but does face the usual additional deviation cost δ for its short-run deviation.

Note that in both ISO optimization problems (1, 2) we have dispensed with non-negativity constraints on the pre-dispatch and dispatch quantity respectively. We will demonstrate that the resulting symmetric equilibria of our NZTS market model will always have associated non-negative pre-dispatch and dispatch quantities. We have eliminated the non-negativity constraints following the convention of supply function equilibrium models (see e.g. Klemperer and Meyer (1989a), Bolle (1992)) in order to enable the analytic computation of equilibrium supply offers.

Firm i's profit in scenario s in this market is then given by

$$u_{i,s}^{TS}(q_i, x_{i,s}) = p_s(y_{i,s}) - \left(\alpha y_{i,s} + \frac{\beta}{2} y_{i,s}^2 + \frac{\delta}{2}(y_{i,s} - q_i)^2\right). \tag{3}$$

3.2. Equilibrium Analysis of the Deterministic Two Period Market

In this section we will present equilibria of the NZTS market model. We will first compute the optimal dispatch quantities from the ISO's optimal dispatch problems (1) and (2) for any number of players. We will then embed these quantities in each generator's expected profit function and allow the generators to simultaneously optimize over their (linear) supply function parameters to obtain equilibrium offers.

PROPOSITION 1. *Problem (1) is a convex program with a strictly convex objective. Its unique optimal solution and the corresponding optimal dual f are given by*

$$f = \frac{Y + ZA}{ZB + 1}$$
$$q_i = fB_i - A_i$$

where $A_i = \frac{a_i}{b_i}$, $B_i = \frac{1}{b_i}$, $A = \sum_{i=1}^n A_i$ and $B = \sum_{i=1}^n B_i$.

Proof. Note that problem (1) has a single linear constraint and that its objective is a strictly convex quadratic as we have assumed that $b_i > 0$ and $Z > 0$. The problem therefore has a unique optimal solution delivered by the first order conditions provided below.

$$Q - \sum_i q_i = 0 \tag{4}$$
$$f - Y + ZQ = 0 \tag{5}$$
$$-f + a_i + b_i q_i = 0 \quad \forall i \tag{6}$$

Algebraic manipulation of equations (5 – 6) will provide the results.

We note that this is similar to Green's analysis in (Green (1996)), although we allow for an intercept parameter as well.

PROPOSITION 2. *For each scenario s, problem (2) is a convex program with a strictly convex objective. Its unique optimal solution and the corresponding optimal dual p_s, are given by*

$$p_s = \frac{Y_s + ZA}{ZB + 1}$$
$$y_{i,s} = p_s B_i - A_i$$

where A_i, B_i, A and B are defined above in proposition (1).

Proof. Problems (2) and (1) are structurally identical, therefore the simple proof of proposition (1) applies again here.

REMARK 1. Note from the above that the pre-dispatch price (and quantity) are equal to the expected spot market prices (and quantities respectively). That is

$$f = \sum_{s=1}^{S} \theta_s p_s. \tag{7}$$

We will now compute the linear supply functions resulting from the equilibrium of the TS market game laid out in (1). Before we begin with the firm computations, we will establish a technical lemma that we utilize in establishing the equilibrium results.

LEMMA 1. *Assume that function $f(x,y): R^2 \to R$ is defined on a domain $D_x \times D_y$ with $D_x, D_y \subseteq R$. Furthermore assume that $x^*(y) \in D_x$, maximizes $f(x,y)$ for any arbitrary but fixed y. Also assume $g(y) = f(x^*(y), y)$ is maximized at $y^* \in D_y$. Then, $f(x,y)$ is maximized at $(x^*(y^*), y^*)$.*

3.2.1. Firm i's computations In this section we will focus on firm i's expected profit function. Note that using equation (7) we obtain

$$u_i^{TS} = E_s[u_{i,s}^{TS}] = \sum_{s=1}^{S} \theta_s \left(p_s y_{i,s} - \left(\alpha y_{i,s} + \frac{\beta}{2} y_{i,s}^2 + \frac{\delta}{2} \left(y_{i,s} - q_i\right)^2 \right) \right).$$

Using propositions (1) and (2), we can re-write u_i^{TS} as a function of a_i and b_i. In order to find a maximum of u_i^{TS} (for a fixed set of competitor offers) we appeal to a transformation that will yield concavity results for u_i^{TS}. We consider u_i^{TS} to be a function of A_i and B_i (instead of a_i and b_i). Note that the transformation ($A_i = \frac{a_i}{b_i}, \quad B_i = \frac{1}{b_i}$) is a one-to-one transformation.

PROPOSITION 3. *Let all competitor (linear) supply function offers be fixed. The following maximizes u_i^{TS} (and is therefore firm i's best response).*

$$\begin{aligned} B_i &= \frac{1 + ZB_{-i}}{Z + \beta + \delta + Z(\beta + \delta)B_{-i}} \\ A_i &= \frac{\alpha + B_i \left(Z\alpha - \delta\left(Y + ZA_{-i}\right)\right) + Z\alpha B_{-i}}{2Z + \beta + Z\beta B_{-i}} \end{aligned}$$

where $A_{-i} = \sum_{j \neq i} A_j$ and $B_{-i} = \sum_{j \neq i} B_j$.

Proof. We can show that u_i^{TS} is a concave function of A_i, assuming B_i is a fixed parameter. Here we have dispensed with the expression for u_i^{TS} as a function of A_i and B_i as it is long and rather complicated. This expression can be found in the online technical companion Khazaei et al. (2013a). We note that u_i^{TS} is a smooth function of A_i and B_i and that

$$\frac{\partial^2 u_i^{TS}}{\partial {A_i}^2} = -\frac{(1+ZB_{-i})\,(2Z+\beta+Z\beta B_{-i})}{(1+ZB)^{\,2}} \leq 0$$

Let B_i be arbitrary but fixed. As u_i^{TS} is a concave function of A_i the first order condition yields an expression for $A_i^*(B_i)$, the value of A_i that maximizes u_i^{TS} (for the fixed B_i).

$$A_i^*(B_i) = \frac{\left(1+ZB_{-i}\right)\left(-Y+\alpha-ZA_{-i}+Z\alpha B_{-i}\right)+B_i\left(Z\left(Y+ZA_{-i}\right)+\left(Z\alpha+\beta Y+Z\beta A_{-i}\right)\left(ZB_{-i}+1\right)\right)}{\left(1+ZB_{-i}\right)\left(2Z+\beta+Z\beta B_{-i}\right)}.$$

We can embed $A_i^*(B_i)$ into u_i^{TS} and find the maximizer in terms of B_i. Lemma (1) then can be applied to demonstrate that the end result delivers the maximum of u_i^{TS}.

After embedding this value of A_i^* into the profit function, the derivative with respect to B_i of u_i^{TS} is

$$\frac{du_i}{dB_i} = \frac{(Y^2 - \sum_s \theta_s Y_s^2)\,(-1+(Z+\beta+\delta)B_i + Z\,(-1+(\beta+\delta)B_i)\,B_{-i})}{(1+ZB)^{\,3}}.$$

$B_i^* = \frac{1+ZB_{-i}}{Z+\beta+\delta+Z(\beta+\delta)B_{-i}}$, is the zero of this derivative. Recall that $Y = \sum_s \theta_s Y_s$, therefore Jensen's inequality implies $Y^2 - \sum_s \theta_s Y_s^2 \leq 0$. Thus, $\frac{du_i}{dB_i} \geq 0$, when $B_i < {B_i}^*$, and $\frac{du_i}{dB_i} \leq 0$, when $B_i > B_i^*$. In other words, u_i is a quasi-concave function of B_i and is maximized at $B_i = {B_i}^*$.

Note that evaluating A_i^* at B_i^* yields

$$A_i^* = \frac{\alpha + B_i\,(Z\alpha - \delta\,(Y+ZA_{-i})) + Z\alpha B_{-i}}{2Z+\beta+Z\beta B_{-i}}.$$

From the above, we can obtain the equilibrium of the NZTS model by solving all best responses simultaneously. This gives the unique and symmetric solution

$$\text{2S-EQM: } B_i = \frac{2}{-(n-2)Z+\beta+\delta+\sqrt{(n-2)^2Z^2+2nZ(\beta+\delta)+(\beta+\delta)^2}} \tag{8}$$

$$A_i = \frac{\alpha+(nZ\alpha - Y\delta)B_i}{2Z+\beta+(n-1)Z(\beta+\delta)B_i}, \tag{9}$$

or alternatively

$$\text{2S-EQM: } b_i = \frac{-(n-2)Z+\beta+\delta+\sqrt{(n-2)^2Z^2+2nZ(\beta+\delta)+(\beta+\delta)^2}}{2} \tag{10}$$

$$a_i = \frac{\alpha b_i + (nZ\alpha - Y\delta)}{2Zb_i+\beta b_i+(n-1)Z(\beta+\delta)}, \tag{11}$$

As we discussed earlier, these equilibrium offers yield non-negative pre-dispatch and dispatch quantities. Below we formally state this result, however the computations to show the non-negativity of these quantities can be found in the technical companion Khazaei et al. (2013a).

PROPOSITION 4. *The equilibrium pre-dispatch and spot production quantities of the firms in the NZTS market are non-negative, i.e.*$q_i \geq 0 \quad \forall i$, *and* $y_{i,s} \geq 0 \quad \forall i,s$ *where* q_i *and* $y_{i,s}$ *are the optimal solutions to problems (1) and (2) respectively using the equilibrium parameters from (10) and (11).*

Proof. For the proof please consult the technical companion Khazaei et al. (2013a).

4. Stochastic Settlement Market

4.1. ISOSP Model

We now introduce the market model we will use to analyze a stochastic settlement market. As discussed in the introduction, the stochastic settlement market contains only a single stage of bidding, but the market clearing procedure takes into account the distribution of future demand when determining dispatch. The market works as follows. When the market opens, demand is uncertain. Firms are allowed to bid their 'normal' cost functions (the cost of producing a given output most efficiently) and a 'penalty' cost function that they would need to be paid to deviate in the short-run. Since firms have quadratic cost functions, they can bid their actual costs by submitting a linear supply function. Each firm i chooses a_i and b_i to bid the linear supply function $a_i + b_i q$, and d_i to bid the (marginal) penalty cost $d_i q$. For a marginal deviation penalty, we look for a function that is zero when the dispatch is not changed from the pre-dispatch quantity, as well as positive to the right and negative to the left of the pre-dispatch. One of the simplest form this function can take is the linear form we have assumed. While the true marginal cost of deviation for a station may be non-linear, it is expected to be smooth as it relates to engineering attributes such as flow of water through an aperture. Therefore to the first order, it can be approximated by a linear function. Note that as with the NZTS model, these bids (a_i, b_i, d_i) need not be their true values (α, β, δ). The offered b_i is required to be positive and d_i should be non-negative.

After generators have placed their bids, the ISO computes the market dispatch according to the stochastic settlement model (outlined below). At this point demand is still uncertain. The ISO chooses two key variables. The first is the *pre-dispatch* quantity for each firm. This is the quantity the ISO asks each firm to prepare to produce, namely the pre-dispatch quantities q_i defined in Section 2. The second is the *short-run deviation* for generator i under each scenario s. This deviation is the variable $x_{i,s}$ defined in Section 2, representing the adjustment made to firm i's predispatch quantity in scenario s. The ISO can choose both pre-dispatch and short-run deviations simultaneously, while aiming to maximize expected social welfare. The ISO assumes that generators have bid their true costs.

In the final stage, demand is realized, and the ISO will ask generators to modify their pre-dispatch quantity according to the short-run deviation for the particular scenario. Each generator ends up with producing $q_i + x_{i,s}$. Two prices are calculated during the course of optimizing welfare. The first is the (shadow) price of the pre-dispatch quantities. We will denote this by f. The second are the prices of each of the deviations, for each of the scenarios. We will denote these by p_s for scenario s. Each generator is paid f per unit for its pre-dispatch quantity q_i, and p_s for its deviations $x_{i,s}$. Thus in realization s, generator i makes profit equal to

$$u_{i,s}^{SS}\left(q_i, x_{i,s}\right) = f q_i + p_s x_{i,s} - \left(\alpha\left(q_{i,s} + x_{i,s}\right) + \frac{\beta}{2}\left(q_{i,s} + x_{i,s}\right)^2 + \frac{\delta}{2} x_{i,s}^2\right). \tag{12}$$

Mathematically, the stochastic optimization problem solved by the ISO can be represented as follows. [5]

ISOSP:

$$\begin{array}{lll} \min\, z = & \sum_{s=1}^{S} \theta_s \left(\sum_{i=1}^{n} \left[a_i(q_i + x_{i,s}) + \frac{b_i}{2}(q_i + x_{i,s})^2 + \frac{d_i}{2} x_{i,s}^2\right] - (Y_s C_s - \frac{Z}{2} C_s^2)\right) & \\ s.t. & \sum_i q_i - Q = 0 & [f] \\ & Q + \sum_i x_{i,s} - C_s = 0 \quad \forall s \in \{1, \ldots, S\} & [\theta_s p_s] \end{array}$$

Q and C_s stand for the total contracted (or pre-dispatched) quantity and total consumption in scenario s respectively. Note that we could have eliminated the two equality constraints. However, their dual variables are the market prices f and p_s respectively, so for clarity we have left them in.

Note that in the ISOSP, an estimate of a future distribution is used. Khazaei et al. (2013b) examine the effectiveness of ISOSP in an empirical competitive setting.

4.2. Characteristics of the Stochastic Optimization Problem

We begin by presenting a series of results that simplify the set of solutions to the ISOSP problem. We start by establishing technical lemmas that enable us to prove that out ISOSP is equivalent to a two period market clearing mechanism similar to NZTS, with the essential difference that now a deviation penalty is present in the ISO's dispatch in real time. These results drastically simplify the subsequent analysis of firms' behaviour in equilibrium.

LEMMA 2. *In the stochastic settlement market clearing, the expected deviation of firm i from pre-dispatch quantity q_i^* is zero, that is, the optimal solution to ISOSP will always satisfy*

$$\sum_s \theta_s x_{i,s}^* = 0.$$

[5] This is a modified version of Pritchard et al.'s problem. There is only one node and thus no transmission constraints, and demand is elastic.

Proof. Let us assume q_i^* and $x_{i,s}^*$ form ISOSP's optimal solution. Let us define for each i and s the quantity $k_{i,s} = q_i^* + x_{i,s}^*$, the total production of firm i in scenario s. Note that $C_s = \sum_i q_i^* + \sum_i x_{i,s}^*$. Assume, on the contrary, that there exists at least one firm j such that $\sum_s \theta_s x_{j,s}^* \neq 0$. The optimal objective value of ISOSP is then given by

$$\sum_i \sum_s \theta_s (a_i k_{i,s} + \frac{b_i}{2}(k_{i,s})^2) + \sum_i \sum_s \theta_s \frac{d_i}{2}(x_{i,s}^*)^2 + Y_s \sum_i k_{i,s} - \frac{Z}{2}(\sum_i k_{i,s})^2. \tag{13}$$

Note that as $\sum_s \theta_s x_{j,s}^* \neq 0$, the term $\sum_i \sum_s \theta_s \frac{d_i}{2}(x_{i,s}^*)^2$ is positive. Now, for a fixed i and $k_{i,s}$ given from above, consider the problem

$$\begin{aligned} \min_{q_i, x_{i,s}} w &= \frac{d_i}{2} \sum_{s=1}^{S} \theta_s x_{i,s}^2 \\ q_i + x_{i,s} &= k_{i,s} \qquad \forall s. \end{aligned} \tag{14}$$

This problem clearly reduces to the univariate problem

$$\min_{q_i} w = \sum_{s=1}^{S} \theta_s (k_{i,s} - q_i)^2,$$

which is optimized at

$$q_i = \sum_{s=1}^{S} \theta_s k_{i,s}.$$

Define $\hat{q_i}$ and $\hat{x_{i,s}}$ by

$$\hat{q_i} = \begin{cases} q_i^*, & i \neq j \\ \sum_{s=1}^{S} \theta_s k_{j,s} & \text{otherwise,} \end{cases}$$

and

$$\hat{x_{i,s}} = \begin{cases} x_{i,s}^*, & i \neq j \\ k_{j,s} - \hat{q_j} & \text{otherwise.} \end{cases}$$

By definition, $\hat{q_i} + \hat{x_{i,s}} = q_i^* + x_{i,s}^*$ for all i and s. It is easy to see that the quantities $\hat{q_i}$ and $\hat{x_{i,s}}$ yield a feasible solution to ISOSP satisfying (14). Furthermore, the objective function evaluated at $\hat{q_i}$ and $\hat{x_{i,s}}$ is given by

$$\sum_i \sum_s \theta_s (a_i k_{i,s} + \frac{b_i}{2}(k_{i,s})^2) + Y_s \sum_i k_{i,s} - \frac{Z}{2}(\sum_i k_{i,s})^2.$$

This value is strictly less than the objective evaluated at q_i^* and $x_{i,s}^*$ (given by 13), as we have already established that $\sum_i \sum_s \theta_s \frac{d_i}{2}(x_{i,s}^*)^2 > 0$. This yields the contradiction that proves the result.

Corollary 1. *In the stochastic problem ISOSP, if $q_i^* + x_{i,s}^* \geq 0$ is satisfied $\forall s \in \{1, \ldots, S\}$ then $q_i^* \geq 0$ will hold.*

Proof. In Lemma 2 we established that $\sum_s \theta_s x^*_{i,s} = 0$. Therefore there exists a scenario s' such that $x^*_{i,s'} \leq 0$. Clearly then $q^*_i + x^*_{i,s'} \geq 0$ implies $q^*_i \geq 0$.

Discussion Lemma 2 is the crucial result that drives the rest of our characterizations. This result hinges on the fact that we penalize quadratic deviation from the pre-dispatch quantity. In the proof of Lemma 2, we demonstrate that the second stage of ISOSP reduces to selecting a contract point that minimizes the quadratic deviation penalty function which is known to be the mean of any distribution. For the quadratic penalty this is irrespective of the distribution. (It is possible to also use an absolute value based deviation penalty and require a symmetric demand distribution. When the penalty function is an absolute value, the point of best estimate is the median of the distribution. For a symmetric distribution of course this reduces again to the mean.) This model penalizes the deviations upward and downward identically. Therefore the predispatch point is optimized based on the mean demand scenario. The reader may argue that allowing for different upward and downward penalties is more realistic. However as Pritchard et al. (2010) show, such allowance of asymmetric penalties can lead to systematic arbitrage by the ISO, where a generator may be required to deviate upward "in every scenario" simply to increase expected welfare. This is undesirable for a market clearing mechanism. We have therefore confined our attention to the symmetric upward and downward penalty case for this paper, which guarantees systematic arbitrage will not occur. We now use the above results and intuition to prove that the ISO's optimization problem can be viewed as a deterministic two period settlement system where unlike NZTS, the deviation penalties are explicitly stated in the ISO's problem in the second period.

LEMMA 3. *The objective function of ISOSP is equivalent to the following function which is separable in the pre-dispatch and the spot market variables*

$$
\begin{aligned}
z = &\sum_{i=1}^{n}\left(a_i q_i + \frac{b_i}{2}q_i^2\right) - Y\sum_{i=1}^{n} q_i + \frac{Z}{2}\left(\sum_{i=1}^{n} q_i\right)^2 \\
&+ \sum_{i=1}^{n}\left(\frac{b_i + d_i}{2}\sum_{s=1}^{S}\theta_s x_{i,s}^2\right) - \sum_{i=1}^{n}\sum_{s=1}^{S}\theta_s Y_s x_{i,s} + \frac{Z}{2}\sum_{s=1}^{S}\theta_s\left(\sum_{i=1}^{n} x_{i,s}\right)^2.
\end{aligned}
$$

Proof. Substituting for C_s from constraints into the objective function of ISOSP yield

$$
\begin{aligned}
z = &\sum_{i=1}^{n}\left(a_i q_i + \frac{b_i}{2}q_i^2\right) - Y\sum_{i=1}^{n} q_i + \frac{Z}{2}\left(\sum_{i=1}^{n} q_i\right)^2 \\
&+ \sum_{i=1}^{n}\left(\frac{b_i + d_i}{2}\sum_{s=1}^{S}\theta_s x_{i,s}^2\right) - \sum_{i=1}^{n}\sum_{s=1}^{S}\theta_s Y_s x_{i,s} + \frac{Z}{2}\sum_{s=1}^{S}\theta_s\left(\sum_{i=1}^{n} x_{i,s}\right)^2 \\
&+ \sum_{i=1}^{n}\left(a_i\sum_{s=1}^{S}\theta_s x_{i,s}\right) + \sum_{i=1}^{n}\left(q_i b_i\sum_{s=1}^{S}\theta_s x_{i,s}\right) + \sum_{s=1}^{S}\theta_s Z\sum_{i=1}^{n}\sum_{j=1}^{n} q_i x_{j,s}
\end{aligned}
$$

We have split the objective in three parts above. Note that the first part of the objective above is exclusively a function of pre-dispatch quantities q_i and the second only a function of the spot dispatches $x_{i,s}$. We rewrite the third segment to make obvious that it is zero at optimality.

$$
\begin{aligned}
z = &\sum_{i=1}^{n}\left(a_i q_i + \frac{b_i}{2}q_i^2\right) - Y\sum_{i=1}^{n} q_i + \frac{Z}{2}\left(\sum_{i=1}^{n} q_i\right)^2 \\
&+\sum_{i=1}^{n}\left(\frac{b_i+d_i}{2}\sum_{s=1}^{S}\theta_s x_{i,s}^2\right) - \sum_{i=1}^{n}\sum_{s=1}^{S}\theta_s Y_s x_{i,s} + \frac{Z}{2}\sum_{s=1}^{S}\theta_s\left(\sum_{i=1}^{n} x_{i,s}\right)^2 \\
&+\sum_{i=1}^{n}\left(a_i\sum_{s=1}^{S}\theta_s x_{i,s}\right) + \sum_{i=1}^{n}\left(q_i b_i \sum_{s=1}^{S}\theta_s x_{i,s}\right) + Z\sum_{i=1}^{n} q_i \sum_{j=1}^{n}\sum_{s=1}^{S}\theta_s x_{j,s}
\end{aligned}
$$

Recall from Lemma (2) that $\sum_{s=1}^{S}\theta_s x_{j,s} = 0$ for the optimal choice of real time dispatches. Therefore we can eliminate the part of the objective. This completes the proof.

Note: We have therefore established that ISOSP reduces to a deterministic two period single settlement model very similar to NZTS but with penalties d_i explicitly present in the second period.

The rest of this section is devoted to deriving explicit expressions for the solution of ISOSP. In the next section we will use these expressions to arrive at best response functions for the firms and subsequently in constructing an equilibrium for the stochastic market settlement. In order to simplify the equations and arrive at explicit solutions, we will transform the space of the parameters of ISOSP (i.e. the firm decision variables), much in the same way that we did in Section 3. If we further define $R_i = \frac{1}{b_i+d_i}$ and $R = \sum_i R_i$, ISOSP reduces to minimizing the following:

$$
\begin{aligned}
z = &\sum_{i=1}^{n}\left(\frac{A_i}{B_i}q_i + \frac{1}{2B_i}q_i^2\right) - Y\sum_{i=1}^{n} q_i + \frac{Z}{2}\left(\sum_{i=1}^{n} q_i\right)^2 \\
&+\sum_{s=1}^{S}\theta_s\left[\sum_{i=1}^{n}\frac{1}{2R_i}x_{i,s}^2 - (Y_s - Y)\sum_{i=1}^{n} x_{i,s} + \frac{Z}{2}\left(\sum_{i=1}^{n} x_{i,s}\right)^2\right].
\end{aligned}
$$

Note that as before (Lemma (3)), the problem is separable in q_i's and $x_{i,s}$'s, we can therefore solve the two stages separately. Note also that the problem in each stage is a convex optimization problem, therefore the first order conditions will readily produce the optimal solution.

Proposition 5. *If (q,x,f,p) represents the solution of ISOSP, then we have*

$$
\begin{aligned}
q_i &= \frac{(Y+ZA)B_i}{1+ZB} - A_i && (15)\\
x_{i,s} &= \frac{(Y_s - Y)R_i}{1+ZR} && (16)\\
f &= \frac{Y+ZA}{1+ZB} \\
p_s &= \frac{Y+ZA}{1+ZB} + \frac{Y_s - Y}{1+ZR}
\end{aligned}
$$

Proof. For derivation of the expressions for the optimal solution above from first order conditions please refer to the technical companion Khazaei et al. (2013a).

Observe from the expression for f that this forward price (paid on pre-dispatch quantities) is independent of any deviation costs in the spot market.

COROLLARY 2. *In the solution of ISOSP, forward price is equal to the expected spot market price.*

Proof. This is simply observed from proposition 5. Note that $Y = \sum_s \theta_s Y_s$.

The fact that the "contract price" f is equal to the expected spot market price, implies that there is no systematic arbitrage.

4.3. Equilibrium Analysis of the Stochastic Settlement Market

In Section (4.1) we presented firm i's profit under scenario s in equation (12). In our market model, we assume that all firms are risk neutral and therefore interested only in maximizing their expected profit. Firm i's expected profit is given by

$$u_i = fq_i + \sum_{s=1}^{S} \theta_s \left(p_s x_{i,s} - \left(\alpha(q_i + x_{i,s}) + \frac{\beta}{2} \ (q_i + x_{i,s})^2 + \frac{\delta}{2} x_{i,s}^2 \right) \right), \tag{17}$$

The above expression for u_i can be expanded and we can observe that

$$\begin{aligned} u_i &= fq_i - (\alpha q_i + \frac{\beta}{2} q_i^2) \\ &+ \sum_{s=1}^{S} \theta_s \left(p_s x_{i,s} - \frac{\beta + \delta}{2} x_{i,s}^2 \right) \\ &- \alpha \sum_{s=1}^{S} \theta_s x_{i,s} - \beta q_i \sum_{s=1}^{S} \theta_s x_{i,s} \end{aligned}$$

Note that from Lemma (2), the generator would know that for any admissible bid, the corresponding expected deviation from pre-dispatch quantities $\sum_{s=1}^{S} \theta_s x_{i,s} = 0$. Therefore the expected profit for the generator becomes

$$\begin{aligned} u_i &= fq_i - (\alpha q_i + \frac{\beta}{2} q_i^2) \\ &+ \sum_{s=1}^{S} \theta_s \left(p_s x_{i,s} - \frac{\beta + \delta}{2} x_{i,s}^2 \right). \end{aligned}$$

We can use the expressions obtained from proposition (5) to write u_i as follows.

$$\begin{aligned} u_i &= -\frac{1}{2}\beta A_i^2 + \frac{A_i \left(-ZA + \alpha + ZB\alpha + ZA\beta B_i + Y\left(-1 + \beta B_i\right)\right)}{1 + ZB} \\ &+ \frac{1}{2(1+ZB)^2(1+ZR)^2} \Bigg(\\ &2(1+ZR)^2(ZA+Y)(ZA+Y-(1+ZB)\alpha)B_i - (1+ZR)^2(ZA+Y)^2\beta B_i^2 \\ &+ (1+ZB)^2 R_i \left(-2 + (\beta+\delta)R_i\right) \left(Y^2 - \sum_s \theta_s Y_s^2 \right) \Bigg) \end{aligned} \tag{18}$$

Although this expression of the expected profit for the generator is rather ugly, it does have the advantage that upon differentiating with respect to R_i, all dependence on A_i and B_i drops and we are left with

$$\frac{du_i}{dR_i} = \frac{(Y^2 - \sum_s \theta_s Y_s^2)\left(-1 + (Z+\beta+\delta)R_i + ZR_{-i}\left(-1 + (\beta+\delta)R_i\right)\right)}{(1+ZR)^3}. \tag{19}$$

Recall that $R_{-i} = \sum_{j \neq i} R_j$. For verification of this derivative term see the technical companion Khazaei et al. (2013a). The fact that this derivative is free of A_i and B_i indicates that u_i is separable in R_i and (A_i, B_i), that is

$$u_i(A_i, B_i, R_i) = g_i(A_i, B_i) + h_i(R_i). \tag{20}$$

Due to this natural separability, our equilibrium analysis will focus on finding best responses in terms of A_i, R_i and B_i, very similar to the NZTS section.

Equation (20) enables us to maximize u_i by maximizing g_i and h_i over (A_i, B_i) and R_i respectively. This is helpful as we can establish quasi-concavity results for g_i and h_i separately.

We start our investigations by examining g_i. The full expression for g_i can be found in the technical companion Khazaei et al. (2013a). Holding B_i fixed, note that

$$\frac{d^2 g_i}{dA_i^2} = -\frac{(1+ZB_{-i})(2Z+\beta+Z\beta B_{-i})}{(1+ZB)^2}.$$

This demonstrates that g_i is concave in A_i for any fixed B_i. Furthermore, for any fixed B_i, we can use the first order conditions to find $A_i^*(B_i)$, i.e. the value of A_i that maximizes $g_i(A_i, B_i)$ for the fixed B_i.

$$A_i^*(B_i) = \frac{(1+ZB_{-i})(\alpha - ZA_{-i} + Z\alpha B - Y) + (Y + ZA_{-i})(Z+\beta+Z\beta B_{-i})B_i}{(1+ZB_{-i})(2Z+\beta+Z\beta B_{-i})} \tag{21}$$

To find the optimal value for g_i, we can now appeal to Lemma (1) and substitute the expression for $A_i^*(B_i)$ in $g_i(A_i^*(B_i), B_i)$. Surprisingly, upon undertaking this substitution, it can be observed that $g_i(A_i^*(B_i), B_i)$ is a constant value. Figure 1 depicts g_i.

To uncover the intuition behind this feature of g_i, we can offer the following mathematical explanation. We observe that

$$\frac{dg_i}{dA_i} = \frac{-(1+ZB_{-i})(Y - \alpha + ZA_{-i} + (2Z+\beta)A_i + ZB_{-i}(-\alpha+\beta A_i))}{(1+ZB)^2} + \frac{(Z(Y+\alpha) + Y\beta + Y(Z\alpha + Y\beta)B_{-i} + ZA_{-i}(Z+\beta+Z\beta B_{-i}))B_i}{(1+ZB)^2}$$

and that

$$\frac{dg_i}{dB_i} = -\frac{Y+ZA}{1+ZB} \cdot \frac{dg_i}{dA_i}.$$

Therefore, stationary conditions enforced in A_i will also imply stationarity in B_i.

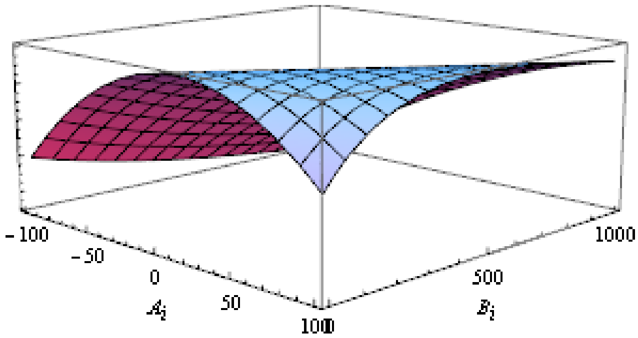

Figure 1 Two views of the function g_i. Note that the optimal value of g_i is obtained along a continuum, for any value of B_i.

As $g_i(A_i^*(B_i), B_i)$ is constant for any $B_i > 0$, for any value of $B_i > 0$, the tuple $(A_i^*(B_i), B_i)$ is an argmax of g_i for any positive B_i.

Let D_i denote $\frac{1}{d_i+\varepsilon}$. Recall that according to our initial assumptions, we have $b_i \geq \varepsilon$, thus

$$\begin{aligned} b_i + d_i \geq \varepsilon + d_i > 0 \Rightarrow \\ \frac{1}{b_i + d_i} \leq \frac{1}{\varepsilon + d_i} \Rightarrow \\ R_i \leq D_i. \end{aligned}$$

The following analysis on h_i will explain how optimal R_i is constrained by the value of D_i.

Proposition 6. *Suppose that R_{-i} is fixed. Then h_i is optimized at*

$$R_i^* = \min\{D_i, \frac{1 + ZR_{-i}}{Z + \beta + \delta + Z(\beta + \delta)R_{-i}}\}$$

Proof. Note that at

$$\hat{R}_i = \frac{1 + ZR_{-i}}{Z + \beta + \delta + Z(\beta + \delta)R_{-i}}, \tag{22}$$

the derivative $\frac{dh_i}{dR_i} = \frac{du_i}{dR_i}$ vanishes. Also recall from Jensen's inequality that $Y^2 \leq \sum_s \theta_s Y_s^2$. It can therefore be seen from (19) that this derivative is positive for $R_i < \hat{R}_i$ and negative for $R_i > \hat{R}_i$. Recall further that the definitions of D_i and R_i require $R_i \leq D_i$. Therefore, in optimizing h_i, we need to enforce this constraint and we obtain

$$R_i^* = \min\{D_i, \frac{1 + ZR_{-i}}{Z + \beta + \delta + Z(\beta + \delta)R_{-i}}\}.$$

We now return to u_i, the expected profit function for firm i. As $u_i(A_i, B_i, R_i) = g_i(A_i, B_i) + h_i(R_i)$, we can start by obtaining the maximum value of g_i attained at a point $(A_i^*(B_i), B_i)$ for any positive B_i. Subsequently, we proceed to optimize $h_i(R_i)$. Proposition (6) readily delivers the optimal R_i. We have therefore proved the following theorem.

THEOREM 1. *The best response of firm i, holding competitor offers fixed, is to offer any d_i for which we have*

$$D_i \geq \frac{1 + ZR_{-i}}{Z + \beta + \delta + Z(\beta + \delta)R_{-i}}.$$

For any such d_i, optimal a_i and b_i can be computed from the following equations.

$$R_i = \frac{1 + ZR_{-i}}{Z + \beta + \delta + Z(\beta + \delta)R_{-i}}$$

$$\frac{1}{B_i} = \frac{1}{R_i} - d_i$$

$$A_i = \frac{(1 + ZB_{-i})(\alpha - ZA_{-i} + Z\alpha B - Y) + (Y + ZA_{-i})(Z + \beta + Z\beta B_{-i})B_i}{(1 + ZB_{-i})(2Z + \beta + Z\beta B_{-i})}$$

Theorem 1 indicates that the game has multiple (infinite) symmetric equilibria. To prevent the problem of unpredictability, caused by multiple equilibria, from here on we assume that the ISO chooses $d_i = d^{SO}$ as a system parameter. This parameter is identical for and known to all participants. This also provides the ISO with the opportunity to choose d^{SO} in a way to obtain a preferable equilibrium (i.e. an equilibrium that yields higher social welfare).

PROPOSITION 7. *The unique symmetric equilibrium quantities of the stochastic settlement market are as follows.*

$$b_i = \max\{\varepsilon, \frac{-Z(n-2) + \beta + \delta + \sqrt{Z^2(n-2)^2 + 2Zn(\beta + \delta) + (\beta + \delta)^2}}{2} - d^{SO}\} \tag{23}$$

$$a_i = \frac{\alpha - Y + B_i(-Z(Y(n-2) - (2n-1)\alpha) + Y\beta + Z(n-1)(Zn\alpha + Y\beta)B_i)}{B_i(Z(n+1) + \beta + Y(n-1)(Zn + \beta)B_i)} \tag{24}$$

The proof of the above proposition is contained in the technical companion Khazaei et al. (2013a). Let us define

$$\hat{d} = \frac{-Z(n-2) + \beta + \delta + \sqrt{Z^2(n-2)^2 + 2Zn(\beta + \delta) + (\beta + \delta)^2}}{2}.$$

THEOREM 2. *In a stochastic settlement market with $d^{SO} \leq \hat{d} - \varepsilon$, as the number of participating firms increases, they tend to offer their true cost parameters. In other words,*

$$\lim_{n \to \infty} a_i = \alpha$$

$$\lim_{n \to \infty} b_i + d^{SO} = \beta + \delta.$$

When the fixed parameter d^{SO} is chosen equal to δ, $\lim_{n \to \infty} b_i = \beta$.

Proof. The equations are simply derived from the equilibrium values of a_i and d_i given in proposition 7.

Theorem 2 shows that the ISOSP market is behaving competitively in the sense that when number of firms increases, they offer their true cost parameters.

One important feature of the equilibrium values are the non-negativity of the pre-dispatch and dispatch. This is important, because we neglected the non-negativity constraints in ISOSP in the first place.

PROPOSITION 8. *Let* $(\mathbf{q}^*, \mathbf{x}^*)$ *represent an equilibrium of the stochastic settlement market, then the following inequalities hold.*

$$\forall i, s : q_i^* + x_{i,s}^* \geq 0$$

$$\forall i : q_i^* \geq 0$$

The proof of the above proposition is contained in the technical companion Khazaei et al. (2013a).

Though, the equilibrium pre-dispatch and dispatch are non-negative, one might raise an objection that a game without the non-negativity constraints embedded in the ISO's optimization problem, is different from the original game. Therefore, there is no assurance the found equilibrium is also the equilibrium of the original game. The following theorem states that the obtained equilibrium values are also the equilibrium of the original game with non-negativity constraints. The proof of this theorem is quite lengthy and consists of several technical lemmas. This proof can be found in the technical companion Khazaei et al. (2013a).

THEOREM 3. *The equilibrium of the symmetric stochastic settlement game without the non-negativity constraints in ISOSP is also the equilibrium of the stochastic settlement game with the non-negativity constraints.*

Proof. Please refer to the technical companion Khazaei et al. (2013a) for the proof of this theorem.

Thus far we established that under the assumption of symmetry, the stochastic settlement (ISOSP) market is equivalent to a two period deterministic settlement in which the deviation penalties are explicitly present in the second period (DTS). We then proceeded to derive an analytical symmetric equilibrium expression for ISOSP. In this process, we enhanced the definition of our game to avoid multiple equilibria and allow the ISO to set a deviation penalty for the (symmetric) players in this game. The issue of multiple equilibria arises as there are multiple optimal solutions to the best response problem. Specifically, for any choice of b_i and d_i, so long as $R_i = \frac{1}{b_i+d_i} = \frac{1}{\hat{d}}$, we obtain an optimal solution (subject to boundary conditions of course).

While in the context of our computations, due to the natural decomposition of u_i, it was natural to treat d_i as the free variable, our intention has been to compare the NZTS mechanism with the ISOSP market clearing proposed. As we observed that ISOSP is equivalent to DTS, it would make sense to think of a game where the ISO imposes the deviation penalty on all participants by choosing $d_i = d^{SO} \geq 0$. If we think of the ISO choosing $0 \leq d^{SO} \leq \hat{d}$, announcing d^{SO} to all participants and imposing this value as the deviation penalty in the second stage, the resulting game, along with its symmetric equilibrium, is equivalent to the game where the ISO selects $b^{SO} = \hat{d} - d^{SO}$.

Here observe that if ISO selects $d^{SO} = \delta$, then as the number of participants increases, in the symmetric equilibrium we obtain $b_i \to \beta$ and $a_i \to \alpha$. Furthermore, it is clear that if $d^{SO} = 0$, then the equilibria for NZTS are recovered.

5. Comparison of the Two Markets

We are interested in the performance of the two market clearing mechanisms ISOSP and NZTS, under strategic behaviour. Our criterion for comparing the two models is social welfare. Social welfare is defined as the sum of the consumer and producer welfare and in our market environments this reduces to

$$\begin{aligned}\mathrm{W} = &\sum_{s=1}^{S} \theta_s \left(Y_s \left(\sum_{i=1}^{n} y_{i,s} \right) - \frac{Z}{2} \left(\sum_{i=1}^{n} y_{i,s} \right)^2 \right) \\ &- \sum_{s=1}^{S} \theta_s \left(\sum_{i=1}^{n} \left(\alpha y_{i,s} + \frac{\beta}{2} {y_{i,s}}^2 + \frac{\delta}{2} (y_{i,s} - q_i)^2 \right) \right). \end{aligned} \tag{25}$$

Note that the different social welfare values W^{SS} (for the ISOSP) and W^{NZTS} (for the NZTS mechanism) are found through the same formula, however with the different equilibrium $y_{i,s}$ variables.

Recall that following Theorem 1 the choice of d^{SO} was delegated to the ISO. The next theorem establishes that when firms are bidding strategically, the stochastic settlement market dominates the NZTS market for any choice of $d^{SO} \in (0, \hat{d})$.

THEOREM 4. *The social welfare of ISOSP is higher than the NZTS market provided the parameter d_i is chosen less than or equal to the threshold value $\hat{d} - \varepsilon$.*

Proof. To prove the proposition, we show when $d^{SO} = 0$, then $\mathrm{W}^{\mathrm{SS}} = \mathrm{W}^{\mathrm{NZTS}}$. We then demonstrate W^{SS} is a increasing function of d^{SO} for $0 \leq d^{SO} \leq \hat{d}$, and therefore, $\mathrm{W}^{\mathrm{SS}} \geq \mathrm{W}^{\mathrm{NZTS}}$, when $d^{SO} \leq \hat{d}$ (note that $\mathrm{W}^{\mathrm{NZTS}}$ is a constant and does not change with d^{SO}).

When $d^{SO} = 0$, equations (24), (9), and (8) yield that the equilibrium quantities are identical in the stochastic settlement and deterministic two period settlement markets. That is

$$\begin{aligned} B_i^{\text{SS}} &= B_i^{\text{NZTS}} \\ A_i^{\text{SS}} &= A_i^{\text{NZTS}} \\ R_i^{\text{SS}} &= B_i^{\text{SS}} \end{aligned}$$

Here we can simplify the expressions for $y_{i,s}$ and q_i (from propositions 1, 2, and 5) to obtain

$$\begin{aligned} q_i^{\text{SS}} &= q_i^{\text{NZTS}} = \frac{YB_i - A_i}{1+ZB} \\ y_{i,s}^{\text{SS}} &= y_{i,s}^{\text{NZTS}} = \frac{Y_sB_i - A_i}{1+ZB} \end{aligned}$$

Therefore social welfare of these models (equation 25) are the same provided $b_i = \hat{b}$.

We can rewrite the social welfare expression (25) as

$$W = \sum_{s=1}^{S} \theta_s \left(Y_s \left(\sum_{i=1}^{n} y_{i,s} \right) - \frac{Z}{2} \left(\sum_{i=1}^{n} y_{i,s} \right)^2 - \sum_{i=1}^{n} \left(\alpha y_{i,s} + \frac{\beta}{2} {y_{i,s}}^2 + \frac{\delta}{2} {x_{i,s}}^2 \right) \right). \tag{26}$$

Note that the expression for social welfare is the same for both models and only depends on the corresponding quantities dispatched from each model (i.e. $y_{i,s}^{SS}$ vs $y_{i,s}^{NZTS}$ etc).

Note that, according to the results of proposition 7, for $0 \le d^{SO} \le \hat{d} - \varepsilon$, we have $b_i = \hat{d} - d^{SO}$, and therefore R_i has a constant value of $1/\hat{d}$, while B_i is a function of d^{SO}. Furthermore, note that $x_{i,s}^{SS}$ is independent of d^{SO}, and therefore,

$$\frac{dW^{SS}}{dd^{SO}} = \frac{1}{(\hat{d} - d^{SO})^2} \sum_{i,s} \frac{dW^{SS}}{dy_{i,s}^{SS}} \frac{dy_{i,s}^{SS}}{dB_i}. \tag{27}$$

On the other hand, taking the derivative of $y_{i,s}^{SS}$ with respect to B_i we obtain

$$\frac{dy_{i,s}^{SS}}{dB_i} = \frac{(Y-\alpha)(n-1)Z^2}{(Z+nZ+\beta+(n-1)Z(nZ+\beta)B_i)^2} \ge 0.$$

The right hand side is readily seen to be non-negative as $Y > \alpha$ and $n > 1$.

As $\frac{dy_{i,s}^{SS}}{dB_i}$ is independent of firm i and scenario s (note that B_i is chosen by the ISO and fixed to a single parameter for all firms,) we can re-arrange (28) and obtain

$$\frac{dW^{SS}}{dd^{SO}} = \frac{1}{(\hat{d} - d^{SO})^2} \frac{dy_{i,s}^{SS}}{dB_i} \sum_{i,s} \frac{dW^{SS}}{dy_{i,s}^{SS}}.$$

On the other hand, differentiating (26) yields

$$\frac{dW^{SS}}{dy^{SS}_{i,s}} = \theta_s \left(Y_s - \alpha - (Zn+\beta) y^{SS}_{i,s}\right)$$

Hence,

$$\begin{aligned}\sum_s \frac{dW^{SS}}{dy^{SS}_{i,s}} &= Y - \alpha - (Zn+\beta)\, q^{SS}_i \\ &= \frac{bZ(Y-\alpha)}{Z(n-1)(nZ+\beta)+b((n+1)Z+\beta)} \geq 0.\end{aligned}$$

Therefore we can conclude that,

$$\frac{dW^{SS}}{dd^{SO}} \geq 0.$$

Note that we can easily show that

$$\hat{d} \geq \beta + \delta,$$

and therefore, if the fixed d_i is chosen equal to δ then $W^{\text{SS}} \geq W^{\text{NZTS}}$.

PROPOSITION 9. *The social welfare of ISOSP is maximized if the parameter d^{SO} is chosen equal to the threshold value $\hat{d} - \varepsilon$.*

Proof. We have established (theorem 4) that $\frac{dW^{SS}}{dd^{SO}} \geq 0$, for $0 \leq d^{SO} \leq \hat{d} - \varepsilon$. To prove this proposition, we demonstrate that for $d^{SO} > \hat{d} - \varepsilon$, we have $\frac{dW^{SS}}{dd^{SO}} \leq 0$. Under this condition, according to the equilibrium formulae, we have $b_i = \varepsilon$, and therefore, changing d^{SO}, only modifies the equilibrium value of R_i (and not A_i and B_i). Therefore, we have

$$\frac{dW^{SS}}{dd^{SO}} = -\frac{1}{(\varepsilon + d^{SO})^2} \sum_{i,s} \frac{dW^{SS}}{dy^{SS}_{i,s}} \frac{dy^{SS}_{i,s}}{dR_i} + \frac{dW^{SS}}{dx^{SS}_{i,s}} \frac{dx^{SS}_{i,s}}{dR_i}. \tag{28}$$

Note that q_i is independent of R_i, and hence,

$$\frac{dy^{SS}_{i,s}}{dR_i} = \frac{dx^{SS}_{i,s}}{dR_i} = \frac{Y_s - Y}{(1+nZR_i)^2}.$$

On the other hand, differentiating (26) yields

$$\begin{aligned}\frac{dW^{SS}}{dy^{SS}_{i,s}} &= \theta_s \left(Y_s - \alpha - (Zn+\beta) y^{SS}_{i,s}\right), \\ \frac{dW^{SS}}{dx^{SS}_{i,s}} &= \theta_s \left(-\delta x^{SS}_{i,s}\right).\end{aligned}$$

Therefore, we conclude

$$\begin{aligned}\frac{dW^{SS}}{dd^{SO}} &= -\frac{1}{(\varepsilon+d^{SO})^2}\sum_{i,s}\frac{dW^{SS}}{dy^{SS}_{i,s}}\frac{dy^{SS}_{i,s}}{dR_i}+\frac{dW^{SS}}{dx^{SS}_{i,s}}\frac{dx^{SS}_{i,s}}{dR_i}\\ &= -\frac{1}{(\varepsilon+d^{SO})^2}\sum_{i,s}\frac{\theta_s(Y_s-Y)}{(1+nZR_i)^2}\left(Y_s-\alpha-(Zn+\beta)y^{SS}_{i,s}-\delta x^{SS}_{i,s}\right)\\ &= -\frac{1}{(\varepsilon+d^{SO})^2\,(1+nZR_i)^2}\sum_{i,s}\theta_s(Y_s-Y)\left(Y_s-\alpha-(Zn+\beta)y^{SS}_{i,s}-\delta x^{SS}_{i,s}\right).\end{aligned}$$

In order to prove the theorem, we show $K_i=\sum_s\theta_s(Y_s-Y)\left(Y_s-\alpha-(Zn+\beta)y^{SS}_{i,s}-\delta x^{SS}_{i,s}\right)\geq 0$ for any i. Note that $y^{SS}_{i,s}=q^{SS}_i+x^{SS}_{i,s}$, and thus

$$\begin{aligned}K_i &= \sum_s\theta_s(Y_s-Y)\left(Y_s-\alpha-(Zn+\beta)y^{SS}_{i,s}-\delta x^{SS}_{i,s}\right)\\ &= \sum_s\theta_sY_s^2-Y^2-(\alpha+Znq^{SS}_i+\beta q^{SS}_i)\sum_s\theta_s(Y_s-Y)-(Zn+\beta+\delta)\sum_s\theta_s(Y_s-Y)x^{SS}_{i,s}\end{aligned}$$

Replacing $\sum_s\theta_s(Y_s-Y)$ with zero and inputting the value of $x^{SS}_{i,s}=\frac{(Y_s-Y)R_i}{1+nZR_i}$, we obtain

$$\begin{aligned}K_i &= \sum_s\theta_sY_s^2-Y^2-(Zn+\beta+\delta)\sum_s\theta_s(Y_s-Y)\frac{(Y_s-Y)R_i}{1+nZR_i}\\ &= \left(\sum_s\theta_sY_s^2-Y^2\right)\left(1-\frac{(Zn+\beta+\delta)R_i}{1+nZR_i}\right)\\ &= \left(\sum_s\theta_sY_s^2-Y^2\right)\left(\frac{1-(\beta+\delta)R_i}{1+nZR_i}\right).\end{aligned}$$

As discussed earlier, $\beta+\delta<\hat{d}$, and thus $0<\beta+\delta<\hat{d}\leq d^{SO}<d^{SO}+\varepsilon$. If we multiply this by the positive value of $R_i=\frac{1}{d^{SO}+\varepsilon}$, we obtain $(\beta+\delta)R_i<1$. Thus, we have

$$\frac{1-(\beta+\delta)R_i}{1+nZR_i}>0.$$

Also, according to Jensen's inequality (or based on the non-negativity property of variance), we can conclude $\mathrm{var}(Y_s)=\sum_s\theta_sY_s^2-Y^2\geq 0$. These inequalities indicate that $K_i\geq 0$, and therefore, we can conclude

$$\frac{dW^{SS}}{dd^{SO}}=-\frac{1}{(\varepsilon+d_i)^2\,(1+nZR_i)^2}\sum_i K_i\leq 0.$$

Example 1. Consider a market with two symmetric generators as defined in table 1.

Figure 2 shows how the social welfare of the stochastic settlement mechanism is affected by the choice of d^{SO}. It also demonstrates that for $d^{SO}<\hat{d}$ and even beyond, the stochastic settlement mechanism has a higher equilibrium social welfare in comparison with the NZTS mechanism. Note

Parameter	Value
α, β, δ	$50, 1, 0.5$
Y_1, Y_2, Z	$100, 150, 1$
θ_1, θ_2	$0.5, 0.5$
n	2

Table 1 The market environment for the example

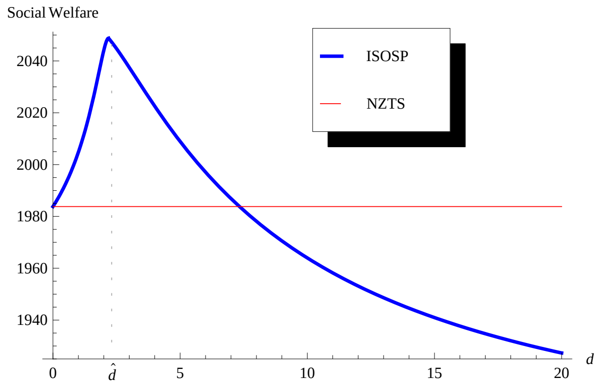

Figure 2 The effect of d^{SO} on the social welfare of the stochastic settlement model and on how it compares to the deterministic two period settlement mechanism.

that at $d^{SO} = \hat{d}$, the equilibrium b_i is set to ε and social welfare is maximized. For the rest of this example, we assume the ISO chooses $d^{SO} = \delta = 0.5$ which ensures higher equilibrium social welfare from the stochastic settlement in comparison with the conventional mechanism.

Another interesting experiment is to investigate the effect of β and δ on these mechanisms.

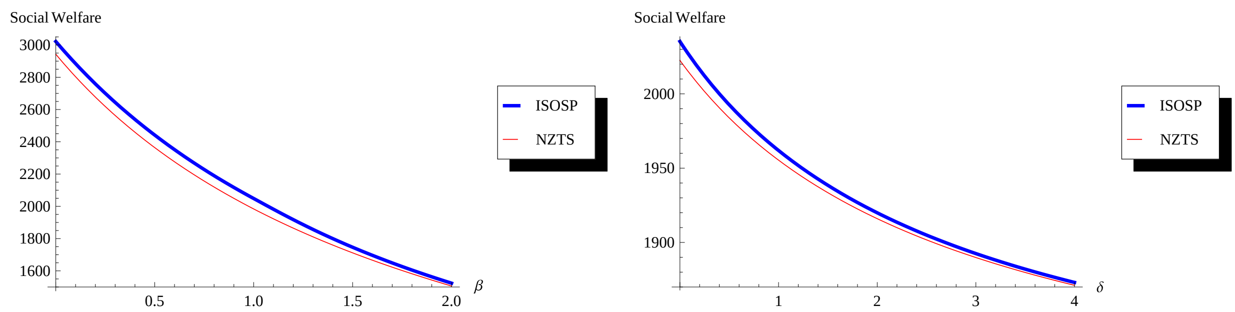

Figure 3 Social welfare of ISOSP and NZTS for different β and δ values.

Figures 3, 4, and 5 compare the ISOSP and the NZTS mechanisms for this example, however for different β and δ values. A first observation is the stochastic settlement mechanism increases social and consumer welfare and decreases producer welfare in comparison with the two settlement mechanism.

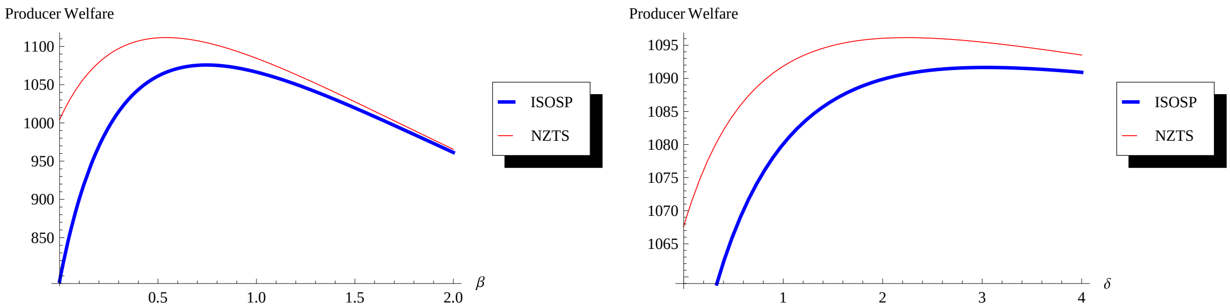

Figure 4 Producer welfare of ISOSP and NZTS for different β and δ values.

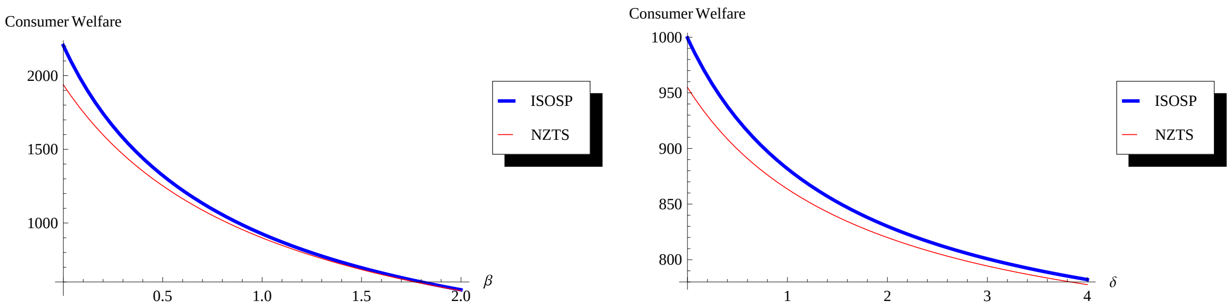

Figure 5 Consumer welfare of ISOSP and NZTS for different β and δ values.

It is also interesting to investigate the effect of competition on these mechanisms. To do so, we can test the effect of number of firms on these mechanism.

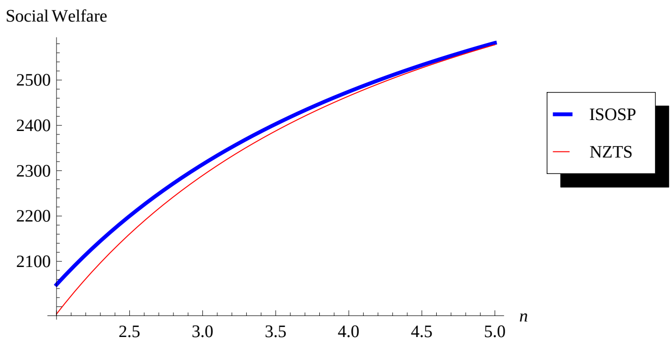

Figure 6 Social welfare of the deterministic two period settlement mechanism converges to that of the stochastic mechanism when n increases. Competition increases with a bigger market.

Figure 6 shows the difference in the social welfare of our two mechanisms as a function of n. It shows that when the number of generators increase, the performance of the stochastic and deterministic two period settlement mechanisms converge.

6. Robustness to Modelling Assumptions

In this section, we investigate the robustness of our results to two important model assumptions.

6.1. The case of asymmetric generators

Thus far, we have derived the analytical expressions for a symmetric equilibrium. We have also proved that the stochastic settlement market always improves social welfare under this symmetric equilibrium. To examine what may happen when participants are not symmetric, we use a computational method laid out in section 6.1.3. We begin this section by laying out our assumptions in the asymmetric case. We will then re-state the ISO's market clearing problem before proceeding to equilibrium computations.

6.1.1. The Market Environment in the Asymmetric Case These assumptions are very similar to the assumptions for the symmetric case with only the assumption of symmetry removed. Below we present the features of the market environment that change due to the asymmetry assumption.

ASSUMPTION 2. *The market environment with asymmetric participants may be defined by the following distinguishing features.*

- *The generators are no longer symmetric. To make the examples computationally tractable, we focus on 2 generator examples.*
- *Each firm i's* long-run *cost function is* $\alpha_i q_i + \frac{\beta_i}{2} q_i^2$, *where* q_i *is the quantity produced by firm i, and* $\beta_i > 0$. *Note that in contrast to the symmetric case, the parameters* α *and* β *are now allowed to be different for different firms.*
- *Each firm's* short-run *cost function is* $\alpha_i (q_i + x_{i,s}) + \frac{\beta_i}{2} (q_i + x_{i,s})^2 + \frac{\delta_i}{2} x_{i,s}^2$, *where* q_i *is the long-run expected dispatch of firm i, and* $q_i + x_{i,s}$ *is the actual short-run dispatch and* $\delta_i > 0$.

6.1.2. Models The ISO's optimization problem is similar to the symmetric case, however theorem (8) no longer applies in the asymmetric case therefore to ensure the non-negativity of the equilibrium, both for the two settlement and the stochastic programming market clearing mechanisms, we need to enforce non-negativity in the ISO's problem.

Therefore, the pre-dispatch problem of the ISO in the (asymmetric) two settlement mechanism is

$$\begin{aligned} \text{PDATS: } \min_{q,Q} z = & \sum_{i=1}^{n} \left(a_i q_i + \frac{b_i}{2} q_i^2 \right) - \left(YQ - \frac{Z}{2} Q^2 \right) \\ s.t. \quad & \sum_i q_i - Q = 0, \qquad [f] \\ & q_i \geq 0, \qquad \forall i. \end{aligned} \tag{29}$$

Also, the ISO's spot market optimization problem for scenario s is

$$\begin{aligned}
\text{SATS(s):} \ \min_{y_s, C_s} z = & \sum_{i=1}^{n} \left(a_i y_{i,s} + \frac{b_i}{2} y_{i,s}^2 \right) - \left(Y_s C_s - \frac{Z}{2} C_s^2 \right) \\
s.t. \quad & \sum_i y_{i,s} - C_s = 0, \qquad [p_s] \\
& y_{i,s} \geq 0, \qquad \forall i.
\end{aligned} \tag{30}$$

Similarly, the stochastic optimization problem of the ISO in the stochastic settlement mechanism can be represented as

$$\begin{aligned}
\text{SPATS:} \ \min z = & \sum_{s=1}^{S} \theta_s \left(\sum_{i=1}^{n} \left[a_i (q_i + x_{i,s}) + \frac{b_i}{2} (q_i + x_{i,s})^2 + \frac{d_i}{2} x_{i,s}^2 \right] - (Y_s C_s - \frac{Z}{2} C_s^2) \right) \\
s.t. \quad & \sum_i q_i - Q = 0, \qquad [f] \\
& Q + \sum_i x_{i,s} - C_s = 0, \qquad \forall s, \qquad [p_s] \\
& q_i + x_{i,s} \geq 0, \qquad \forall i, s.
\end{aligned}$$

6.1.3. Equilibrium computations To find a Nash equilibrium to our games, we use a dynamic process. The idea is to allow each participant in turn to update its strategies, assuming the strategy set of the other participants is fixed. If this diagonalization process terminates with no participant willing to deviate from its last strategy, then we have arrived at a Nash equilibrium.

Diagonalization procedure for the two settlement market clearing

1. Start with an initial supply function for each participant. For our examples we start with the true cost function for each generator.

2. While an equilibrium is not obtained (i.e. there exists at least one generator that has changed its supply function in the last round), solve the best reply problem to (global) optimality.

$$\begin{aligned}
\text{BR[i]:} \ \max_{a_i, b_i} u_i = \ & f q_i + \sum_s \theta_s \Big(p_s x_{i,s} - \alpha_i (q_i + x_{i,s}) - \frac{\beta_i}{2} (q_i + x_{i,s})^2 \\
& - \frac{\delta_i}{2} x_{i,s}^2 \Big) \\
s.t. \quad & \{q_i, f\} \text{ are optimal for PDATS and } \{x_{i,s}, p_s\} \text{ optimal for SATS(s).} \\
& \{a_i, b_i\} \in \Psi_i
\end{aligned}$$

Here

- u_i stands for the expected profit function of firm i which is the difference between its expected income and its expected cost of production.
- Ψ_i indicates the constraints imposed by the regulator on offered supply functions player i i.e. a_i and b_i.

Diagonalization procedure for stochastic programming market clearing

This procedure is almost identical to the diagonalization for the two settlement market clearing except ISO's optimization is a stochastic programming.

1. Start with an initial supply functions for each participant. For our examples we start with the true cost function for each generator.

2. While an equilibrium is not obtained (i.e. there exists at least one generator that has changed its supply function in the last round):

$$\begin{aligned}
\text{BR[i]: } \max_{a_i,b_i} u_i = \quad & f q_i + \sum_s \theta_s \Big(p_s x_{i,s} - \alpha_i (q_i + x_{i,s}) - \frac{\beta_i}{2}(q_i + x_{i,s})^2 \\
& - \frac{\delta_i}{2} x_{i,s}^2 \Big) \\
s.t. \qquad & \{q_i, x_{i,s}, f, p_s\} \text{ are optimal for SPATS.} \\
& \{a_i, d_i\} \in \Psi_i
\end{aligned}$$

Here

- u_i stands for the expected profit function of firm i which is the difference between its expected income and its expected cost of production.
- Ψ_i indicates the constraints imposed by the regulator on offered supply function intercept and deviation cost for player i i.e. a_i and b_i.

To solve the non-convex problem BR[i], in both of the diagonalization procedures above, it is necessary to use global optimization. For solving this optimization problem, we have used the global solver of LINGO. The global solver of LINGO guarantees the optimality of its final solution using a branch and bound approach. Here a sequence of piecewise convex relaxations of the original (non-convex) problem are solved. The convex relaxations are derived using bounds on the variables. If the optimal solution of the relaxed problem is feasible for the original problem, it is also the optimal point of the original problem. If not, further enhancement is made through dividing up the domain of the objective function and creating more accurate, piecewise convex functions on each part of the domain. The process of branching continues until all branches end with an optimal point. Note that user defined tolerances on slitting procedure make this method a finite process. For more information about the mathematics behind this global solver see Lin and Schrage (2009). The tolerance that we have used, as the minimum acceptable difference between best response strategies of firms in different turns, is of order of 10^{-10}. The LINGO code is available in the technical companion Khazaei et al. (2013a).

Scenarios	θ	Y
1	0.5	100
2	0.5	150

, $Z = 1$

Table 2 Scenarios and demand

6.1.4. Sensitivity to different cost structures (i.e. generation technologies) Different generation technologies have different structure in their cost functions, e.g. a particular generation technology may have a high generation cost but a low cost for fast deviation and another generator might be the opposite. In this section, we analyse a market with two asymmetric generators with various cost patterns (different layouts of α_i, β_i, and, δ_i). Without loss of generality we call the generator with the lower α_i value generator 1 and the other generator 2. We then design different experiments with different possibilities for β_i and δ_i (e.g. $\beta_1 > \beta_2$, $\beta_1 < \beta_2$ and etc.). For each of these layouts, we consider two cases for d^{SO}: $d^{SO} = \min_i\{\delta_i\}$ and $d^{SO} = \max_i\{\delta_i\}$. Consider a market with two demand scenarios with parameters given in table 2. Table 3 summarizes the difference between the equilibrium values of the SFSP and NZTS mechanisms for each of these experiments. According to these results, SFSP results in lower prices, profit, and producer welfare, and higher consumer and social welfare in comparison with the NZTS mechanism.

Table 3 Difference in the equilibrium values of the SFSP and NZTS (i.e. SFSP − NZTS) under asymmetry assumption and different cost patters.

Ex.	Gen	α	β	δ	d^{SO}	f	$p(s_1)$	$p(s_2)$	Profit	CW	PW	SW
1	1	0	0.001	0.001	0.001	-0.0057	-0.01	-0.01	-0.19	0.48	-0.30	0.18
	2	10	1	0.5	0.001	-0.0057	-0.01	-0.01	-0.11			0.18
2	1	0	0.001	0.5	0.001	-0.014	-0.01	-0.02	-0.58	1.22	-0.81	0.41
	2	10	1	0.001	0.001	-0.014	-0.01	-0.02	-0.23			0.41
3	1	0	0.001	0.5	0.5	-2.58839	-2.59	-2.59	-67.40	218.43	-119.41	99.02
	2	10	1	0.5	0.5	-2.58839	-2.59	-2.59	-52.00			99.02
4	1	0	1	0.001	0.001	-0.00775	-0.01	-0.01	-0.23	0.64	-0.37	0.27
	2	10	0.001	0.5	0.001	-0.00775	-0.01	-0.01	-0.14			0.27
5	1	0	1	0.5	0.001	-0.02637	-0.03	-0.03	-0.95	2.16	-1.05	1.10
	2	10	0.001	0.001	0.001	-0.02637	-0.03	-0.03	-0.10			1.10
6	1	0	1	0.5	0.5	-2.4747	-2.48	-2.47	-58.07	202.06	-140.63	61.44
	2	10	0.001	0.5	0.5	-2.4747	-2.48	-2.47	-82.56			61.44
7	1	0	0.001	0.001	0.5	-3.55014	-3.55	-3.55	-160.97	305.33	-231.03	74.30
	2	10	1	0.5	0.5	-3.55014	-3.55	-3.55	-70.07			74.30
8	1	0	0.001	0.5	0.5	-3.99094	-3.99	-3.99	-91.25	348.13	-174.46	173.67
	2	10	1	0.001	0.5	-3.99094	-3.99	-3.99	-83.21			173.67
9	1	0	1	0.001	0.5	-3.6365	-3.64	-3.64	-97.94	305.61	-200.01	105.60
	2	10	0.001	0.5	0.5	-3.6365	-3.64	-3.64	-102.07			105.60
10	1	0	1	0.5	0.5	-3.72095	-3.72	-3.72	-89.78	311.02	-282.17	28.86
	2	10	0.001	0.001	0.5	-3.72095	-3.72	-3.72	-192.38			28.86

Gen	α	β	δ
1	0	0.001	0.001
2	10	1	0.5

Table 4 Cost function of generators

Table 5 Equilibrium values of SFSP for different levels of d^{SO} in comparison with the equilibrium values of NZTS

Gen	d^{SO}	a	b	$b+d$	CW	PW	SW
1	0.0001	-0.0569	0.6563	0.6564	3688.9	3092.4	6781.3
2	0.0001	-1.0159	1.8962	1.8963			
1	0.001	-0.0078	0.6554	0.6564	3689.3	3092.1	6781.4
2	0.001	-1.0042	1.8953	1.8963			
1	0.01	0.4858	0.6464	0.6564	3693.3	3089.5	6782.8
2	0.01	-0.8876	1.8863	1.8963			
1	0.1	5.4164	0.5565	0.6565	3735.6	3060.9	6796.5
2	0.1	0.2343	1.7963	1.8963			
1	0.5	27.2341	0.1566	0.6566	3993.3	2861.7	6855.0
2	0.5	3.7615	1.3955	1.8955			
1	2	15.8928	ε	2	6027.3	1782.8	7810.0
2	2	14.9124	0.1667	2.1667			
1	3	15.7103	ε	3	6022.1	1769.4	7791.5
2	3	15.7103	ε	3			
1	10	36.2862	ε	10	3943.7	2692.3	6636.1
2	10	36.2862	ε	10			
1	100	36.2862	ε	100	3935.2	2620.8	6556.0
2	100	36.2862	ε	100			
1	NZTS	-0.0622	0.6567	NA	3688.0	3092.7	6780.7
2	NZTS	-1.0199	1.8964	NA			

6.1.5. Sensitivity to d^{SO} To analyse the sensitivity of our results to the value of d_i, we focus on the first experiment above with the cost parameters listed in table 4. To compare the stochastic settlement mechanism with the two settlement mechanism, we find the equilibrium values of the SFSP mechanism on a range of different d^{SO}s. The equilibrium values of the two settlement mechanism and the stochastic settlement mechanism for different bs are listed in table 5.

This table indicates that our proven results of the symmetric case are expected in this case as well. Firstly, SFSP yields higher social welfare for $d^{SO} \in (0, \max_i\{\delta_i\})$. Secondly, social welfare is increasing with respect to d^{SO} in this range and reaches its climax at a much higher level of d^{SO} (somewhere between 2 and 3 in this example). After this point, social welfare starts to drop with higher d^{SO} values and ends up lower than that of the NZTS mechanism for very large d^{SO}s. The third similarity is that generators submit $b = \varepsilon$ when d^{SO} is larger than a threshold value.

6.2. Restriction to the case of supply functions with intercept zero

The model originally used by Green (1996) restricted the linear supply offers to have intercept at zero. We went through the exercise of constructing NZTS and ISOSP equilibrium results when supply functions comply with the zero intercept rule (note that this eliminates one variable from the decision space of the generators). For this case, we restricted the model to a duopoly. The methodology we have used for this case follows that of the general case. For each market clearing mechanism we establish the values of q_i, f, $x_{i,s}$ and p_s as before. Then we obtain the expected utility expressions and establish quasi-concavity results for each case following the same lines of argument used in the previous sections. (The details are contained in the last section of the technical companion Khazaei et al. (2013a).

When restricted by the zero intercept condition, and responding to the opponent restricted by the same condition, the best reply and hence equilibrium values change. Furthermore, we find that in the ISOSP case, we have a unique (symmetric) equilibrium. An interesting observation here is that allowing one more degree of freedom, by the choice of an intercept, leading to a continuum of equilibria, leads the ISO to acknowledge a deviation cost (penalty) for the participants (in efforts to improve welfare). As we observed, for any choice of deviation penalty in $(0, \delta)$, the welfare of ISOSP is improved over NZTS. This is no longer the case for the Green type linear supply functions. That is, the welfare difference between NZTS and ISOSP can be positive or negative. Specifically, if we fix $\beta = 2.0$, $\delta = 4.0$, and note $W^{\text{NZTS}} - W^{\text{SS}}$ by TWD, then

$Y = 5$	$Y = 1.25$
$\sigma_Y^2 = 0.25$	$\sigma_Y^2 = 0.0625$
$Z = 0.5$	$Z = 0.125$
TWD = -4.26892	**TWD = 1.54577**

7. Conclusion

In this paper, we set up a simple modelling environment in which we were able to compare the New Zealand inspired deterministic two period single settlement market clearing mechanism against a stochastic settlement auction which reduces to another two period single settlement auction with explicit penalties of deviation, therefore different from the NZTS model. We were able to model firms' best responses in these markets, and so find equilibrium behaviour in each. We find that in our symmetric models, the ISOSP auction provably dominates the NZTS auction when measuring expected social welfare.

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Technical Companion for the Paper "Single and Multi-settlement Approaches to Market Clearing Mechanisms under Demand Uncertainty"

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1 Computations for Proposition 3

In[1]:= $$c_{s\_} = \frac{Y_s\, cB - cA}{Z\, cB + 1};$$

$$p_{s\_} = \frac{(Y_s + Z\, cA)}{Z\, cB + 1};$$

$$y_{i\_,s\_} = p_s\, B_i - A_i;$$

$$Q = \frac{EY\, cB - cA}{Z\, cB + 1};$$

$$f = \frac{(EY + Z\, cA)}{Z\, cB + 1};$$

$$q_{i\_} = f\, B_i - A_i;$$

$$cA = A_i + A_{-i};$$

$$cB = n\, B_i;$$

$$u_{i\_} = \text{FullSimplify}\left[\sum_{s=1}^{S} \theta_s \left(p_s\, y_{i,s} - \left(\alpha\, y_{i,s} + \frac{\beta}{2}\, y_{i,s}{}^2 + \frac{\delta}{2}\, \left(y_{i,s} - q_i\right)^2\right)\right)\right];$$

In[11]:= $$u_i = \text{FullSimplify}[u_i]$$

Out[11]= $$\sum_{s=1}^{S} \left(-\alpha \left(-A_i + \frac{B_i\, (Z\, (A_{-i} + A_i) + Y_s)}{1 + n\, Z\, B_i}\right) + \frac{(Z\, (A_{-i} + A_i) + Y_s)\, \left(-A_i + \frac{B_i\, (Z\, (A_{-i}+A_i)+Y_s)}{1+n\, Z\, B_i}\right)}{1 + n\, Z\, B_i} - \frac{1}{2}\, \beta \left(-A_i + \frac{B_i\, (Z\, (A_{-i} + A_i) + Y_s)}{1 + n\, Z\, B_i}\right)^2 - \frac{1}{2}\, \delta \left(-\frac{(EY + Z\, (A_{-i} + A_i))\, B_i}{1 + n\, Z\, B_i} + \frac{B_i\, (Z\, (A_{-i} + A_i) + Y_s)}{1 + n\, Z\, B_i}\right)^2\right) \theta_s$$

In[12]:= $$EY = \sum_{s=1}^{S} \theta_s\, Y_s$$

Out[12]= $$\sum_{s=1}^{S} Y_s\, \theta_s$$

$$\text{FullSimplify}[D[u_i, A_i, A_i]]$$

Out[23]= $$-\frac{(1 + (-1 + n)\, Z\, B_i)\, (2\, Z + \beta + (-1 + n)\, Z\, \beta\, B_i)}{(1 + n\, Z\, B_i)^2}$$

2 Proposition 4

Proposition. *The equilibrium pre-dispatch and spot production quantities of the firms in the NZTS market are non-negative, i.e.*$q_i \geq 0 \quad \forall i$, *and* $y_{i,s} \geq 0 \quad \forall i, s$ *where* q_i *and* $y_{i,s}$ *are the optimal solutions to problems (1) and (2) respectively using the equilibrium parameters from (10) and (11).*

Proof. To prove the proposition, we first show the equilibrium price intercept of the supply function of generators (i.e. $a_i = \frac{A_i}{B_i}$) is less than the price intercept of the demand function (i.e. Y and Y_s). Then, we show this property entails the non-negativity of equilibrium quantities.

Substituting A_i and B_i from proposition 3.4 into $a_i = \frac{A_i}{B_i}$, and then taking the derivative of a_i with respect to Z, we achieve

$$\frac{\partial a_i}{\partial Z} = \frac{2\delta\left((n-2)^2 Z + 2k + n(\beta+\delta+k)\right)(Y-\alpha)}{k((n+2)Z+\beta-\delta+k)^2},$$

where, $k = \sqrt{(n-2)^2 Z^2 + 2nZ(\beta+\delta) + (\beta+\delta)^2}$. Because $n \geq 2$, $Z > 0$, $\beta \geq 0$, $\delta \geq 0$, and $\alpha \leq Y$, we have

$$\frac{\partial a_i}{\partial Z} \geq 0. \tag{1}$$

On the other hand, taking the limit of a_i as Z approaches infinity, we obtain

$$\lim_{Z\to\infty} a_i = \alpha. \tag{2}$$

Equations (1) and (2) yield

$$a_i \leq \alpha.$$

This together with assumption $\alpha \leq Y_s$,$\forall s$ yields

$$a_i \leq Y_s \qquad \forall i, s. \tag{3}$$

Using $a_i = \frac{A_i}{B_i}$, we can rewrite equation (3) as

$$B_i Y_s - A_i \geq 0 \qquad \forall i, s. \tag{4}$$

Also, using the value of B_i from proposition 3.4, we can show $B_i \geq 0$. Thus, we can conclude

$$B \geq 0 \tag{5}$$

On the other hand, embedding p_s into $y_{i,s}$ from proposition 3.2, we obtain

$$y_{i,s} = \frac{B_i Y_s - A_i}{ZB+1} \qquad \forall i, s.$$

This together with equations (4) and (5) and assumption $Z > 0$ gives

$$y_{i,s} \geq 0 \qquad \forall i, s.$$

From propositions 3.1 and 3.2, we achieve $q_i = \sum_s \theta_s y_{i,s}$. As $\theta_s \geq 0$, we obtain

$$q_i \geq 0 \qquad \forall i$$

□

3 The optimal solution to ISOSP problem: proof of Proposition 5

Proposition. *If* $(\mathbf{q}, \mathbf{x}, \mathbf{f}, \mathbf{p})$ *represents the solution of ISOSP, then we have*

$$\begin{aligned} q_i &= \frac{(Y + ZA)B_i}{1 + ZB} - A_i && (6) \\ x_{i,s} &= \frac{(Y_s - Y)R_i}{1 + ZR} && (7) \\ f &= \frac{Y + ZA}{1 + ZB} \\ p_s &= \frac{Y + ZA}{1 + ZB} + \frac{Y_s - Y}{1 + ZR} \end{aligned}$$

Proof. The Lagrangian function of ISOSP can be represented as follows.

$$\begin{aligned} L &= -f\left(-Q + \sum_{i=1}^{n} q_i\right) \\ &+ \sum_{s=1}^{S} \theta_s \Bigg(-p_s \left(Q - C_s + \sum_{i=1}^{n} x_{i,s}\right) \\ &-Y_s C_s + \frac{Z C_s^2}{2} + \sum_{i=1}^{n} \left(\frac{1}{2} d_i x_{i,s}^2 + a_i \left(q_i + x_{i,s}\right) + \frac{1}{2} b_i \left(q_i + x_{i,s}\right)^2\right)\Bigg) \end{aligned}$$

Taking derivative with respect to different variables yields to the following equations.

$$\frac{dL}{dq_i} = -f + \sum_s \theta_s \left(a_i + b_i \left(q_i + x_{i,s}\right)\right) \tag{8}$$

$$\frac{dL}{dx_{i,s}} = \theta_s \left(-p_s + a_i + b_i \left(q_i + x_{i,s}\right) + d_i x_{i,s}\right) \tag{9}$$

$$\frac{dL}{dC_s} = \theta_s\left(p_s - Y_s + ZC_s\right) \tag{10}$$

$$\frac{dL}{dQ} = f - \sum_s \theta_s p_s \tag{11}$$

$$\frac{dL}{dp_s} = \theta_s\left(-Q + C_s - \sum_i x_{i,s}\right) \tag{12}$$

$$\frac{dL}{df} = Q - \sum_i q_i \tag{13}$$

The Lagrangian is evidently a convex function. Thus, for finding the solution of the stochastic program, we should set all above derivatives to zero.

From (8)

$$f = a_i + b_i q_i + \sum_s p_s x_{i,s}. \tag{14}$$

From (9) and (14)

$$p_s = f + (b_i + d_i)x_{i,s}, \tag{15}$$

and from (11)

$$f = \sum_s \theta_s p_s. \tag{16}$$

Now (14), (15) and (16) result in the following conclusion, as it is also concluded from lemma 4.1.

$$\sum_s \theta_s x_{i,s} = 0 \tag{17}$$

(14) and (17) lead to

$$f = a_i + b_i q_i. \tag{18}$$

Consequently, forward price is independent of the spot market and is resolved merely by contract quantities. Though, contract quantities are chosen by considering different possible spot scenarios.

From (10),

$$p_s = Y_s - ZC_s, \tag{19}$$

from (12),

$$C_s = Q + \sum_i x_{i,s}, \tag{20}$$

and from (13),

$$Q = \sum_i q_i \tag{21}$$

can be concluded.
(17) and (20) lead to

$$\sum_s \theta_s C_s = Q. \tag{22}$$

(16), (19) and (22) make the following conclusion.

$$f = Y - ZQ \tag{23}$$

Now from (18) and (23) we can conclude

$$q_i = \frac{Y - ZQ - a_i}{b_i}. \tag{24}$$

In consequence, from (21) and summation of q_i from (24) over all firms and by using the transformation (A_i, B_i, R_i), we obtain

$$Q = (Y - ZQ)B - A.$$

Therefore,

$$Q = \frac{YB - A}{1 + ZB}. \tag{25}$$

Now the following inference can be resulted from (24) and (25).

$$q_i = \frac{(Y + ZA)B_i}{1 + ZB} - A_i \tag{26}$$

Now let us find $x_{i,s}$. (15), (19) and (20) give

$$f + (b_i + d_i)x_{i,s} = Y_s - ZQ - Z\sum_i x_{i,s}.$$

By adding (23) to this equation following equation is resulted.

$$x_{i,s} = \frac{Y_s - Y - Z\sum_i x_{i,s}}{b_i + d_i} \tag{27}$$

Now by getting a summation from (27) and simplifying the resulted equation we achieve

$$\sum_i x_{i,s} = \frac{(Y_s - Y)R}{1 + ZR}.$$

By inserting this equation in (27), we obtain

$$x_{i,s} = \frac{(Y_s - Y)R_i}{1 + ZR}, \tag{28}$$

and from (23) and (25), first stage price can be extracted.

$$f = \frac{Y + ZA}{1 + ZB} \tag{29}$$

One observation about this equation is that contract price is independent of R, in other words, it is independent of deviating cost in the spot market.

(25), (28) and (29) determine spot price for each scenario.

$$p_s = \frac{Y + ZA}{1 + ZB} + \frac{Y_s - Y}{1 + ZR} \tag{30}$$

□

4 The equilibrium of the stochastic settlement market: proof of Proposition 7

Proposition. *The unique symmetric equilibrium quantities of the stochastic settlement market are as follows.*

$$b_i = \max\{\varepsilon, \frac{-Z(n-2) + \beta + \delta + \sqrt{Z^2(n-2)^2 + 2Zn(\beta+\delta) + (\beta+\delta)^2}}{2} - d^{SO}\} \tag{31}$$

$$a_i = \frac{\alpha - Y + B_i\left(-Z(Y(n-2) - (2n-1)\alpha) + Y\beta + Z(n-1)(Zn\alpha + Y\beta)B_i\right)}{B_i\left(Z(n+1) + \beta + Y(n-1)(Zn+\beta)B_i\right)} \tag{32}$$

Proof. To find a symmetric equilibrium, we can use

$$\begin{aligned} A_{-i} &= (n-1)A_i, \\ B_{-i} &= (n-1)B_i, \\ R_{-i} &= (n-1)R_i \\ \frac{1}{R_i} &= \frac{1}{B_i} + d^{SO}. \end{aligned}$$

By putting these equations in the best response functions (from theorem 1) and solving the resulted equations with respect to A_i and B_i, following equilibrium equations is resulted.

$$A_i = \frac{\alpha - Y + B_i\left(-Z(Y(n-2) - (2n-1)\alpha) + Y\beta + Z(n-1)(Zn\alpha + Y\beta)B_i\right)}{Z(n+1) + \beta + Y(n-1)(Zn+\beta)B_i}$$

$$\frac{1}{B_i} = \max\{\varepsilon, \frac{-Z(n-2) + \beta + \delta + \sqrt{Z^2(n-2)^2 + 2Zn(\beta+\delta) + (\beta+\delta)^2}}{2} - d^{SO}\} \tag{33}$$

Let us see why equation (33) implies a true equilibrium quantity. Let $\hat{d} = \frac{-Z(n-2)+\beta+\delta+\sqrt{Z^2(n-2)^2+2Zn(\beta+\delta)+(\beta+\delta)^2}}{2}$ and $\hat{R} = 1/\hat{d}$. If $d^{SO} \leq \hat{d} - \varepsilon$, $b_i = \hat{d} - d^{SO}$ (equivalently $R_i = \hat{R}$) satisfies the best response function for R_i. When $d^{SO} > \hat{d} - \varepsilon$, we must show that the equilibrium b_i is equal to ε (or equivalently equilibrium $R_i = D_i = \frac{1}{\hat{d}+\varepsilon}$). It means when the other generators j have chosen $R_j = D_j$, the best response for the firm i is also to choose $R_i = D_i$, and so we need to show $\frac{1+Z(n-1)D_i}{Z+\beta+\delta+Z(n-1)(\beta+\delta)D_i} \geq D_i$. Note that D_i is a fixed quantity chosen by the ISO, Thus, $D_j = D_i$.

Define $f(x) = \frac{1+Z(n-1)x}{Z+\beta+\delta+Z(n-1)(\beta+\delta)x} - x$. We can easily show that $f(x)$ is a concave function for $x \geq 0$:

$$f''(x) = -\frac{2Z^3(n-1)^2(\beta+\delta)}{(Z+\beta+\delta+Z(n-1)(\beta+\delta)x)^3} < 0$$

Also $f(0) = \frac{1}{Z+\beta+\delta} > 0$ and $f(\hat{R}_i) = 0$. Thus for $0 < D_i < \hat{R}_i$, and by considering concavity of $f(x)$,

$$f(D_i) \geq 0.$$

Therefore,

$$\frac{1+Z(n-1)D_i}{Z+\beta+\delta+Z(n-1)(\beta+\delta)D_i} \geq D_i.$$

□

5 Stochastic settlement yields non-negative equilibria: proof of Proposition 8

Proposition. *If* $(\mathbf{q}^*, \mathbf{x}^*)$ *represents the equilibrium of the stochastic settlement market, following equations always hold.*

$$\forall i, s : \, q_i^* + x_{i,s}^* \geq 0$$

$$\forall i : \, q_i^* \geq 0$$

Proof. From (26) and (28), the following equation can be resulted.

$$y_{i,s} = q_i^* + x_{i,s}^* = \frac{(Y+ZA)B_i}{1+ZB} - A_i + \frac{(Y_s - Y)R_i}{(1+ZR)}$$

It is obvious that if $y_{i,s}$ is non-negative for the scenario that has the lowest Y_s, it is non-negative for the other scenarios as well. Thus, we prove this only for the scenario s' for which we have $Y_{s'} \leq Y_s$ for all s. If we assume having at least two different scenarios with positive probabilities, we have

$$Y_{s'} < Y. \tag{34}$$

Let us first define $\hat{R}_i = \frac{2}{-Z(n-2)+\beta+\delta+\sqrt{Z^2(n-2)^2+2Zn(\beta+\delta)+(\beta+\delta)^2}}$, as we defined in the proof of proposition 4.8. Now consider $y'_{i,s'} = \min_{\alpha,\delta} y_{i,s'}$. Obviously if we prove that $y'_{i,s'}$ is non-negative, we have also proven the non-negativity of $y_{i,s}$. $y_{i,s}$ can be divided to two separate functions of α and δ, such that

$$\frac{dy_{i,s'}}{d\delta} = \begin{cases} if\ \hat{R}_i \leq B_i : \\ \frac{2\left(Zn+\beta+\delta+\sqrt{Z^2(n-2)^2+2Zn(\beta+\delta)+(\beta+\delta)^2}\right)(Y-Y_s)}{\sqrt{Z^2(n-2)^2+2Zn(\beta+\delta)+(\beta+\delta)^2}\left(Z(n+2)+\beta+\delta+\sqrt{Z^2(n-2)^2+2Zn(\beta+\delta)+(\beta+\delta)^2}\right)^2} \\ \\ Otherwise : \\ 0 \end{cases}$$

$$\frac{dy_{i,s'}}{d\alpha} = -\frac{1+ZB_1(n-1)}{Z(n+1)+\beta+ZB_1(n-1)(Zn+\beta)}$$

The parameters Z, β, and δ are non-negative. Thus, from (34), we can conclude

$$\frac{dy_{i,s}}{d\delta} \geq 0,$$

$$\frac{dy_{i,s}}{d\alpha} \leq 0.$$

Consequently, $\delta = 0$ and $\alpha = Y_{s'}$ minimize $y_{i,s'}$. Note that we have assumed in this chapter, that y-intercept of cost function (α) is less than y-intercept of the demand scenarios ($Y_{s'}$). Thus, we prove that $y'_{i,s'} = y_{i,s'}(\delta = 0, \alpha = Y_{s'})$ gets non-negative values.

When $\delta = 0$, at $\hat{\beta} = \frac{1+ZB_i(n-2)}{B_i(1+ZB_i(n-1))}$, we have $\hat{R}_i = B_i$. By applying the fact that $\hat{R}_i$ is a decreasing function of β, we can conclude,

$$R_i = \begin{cases} B_i & \beta < \hat{\beta} \\ \hat{R}_i & \beta \geq \hat{\beta} \end{cases}$$

and

$$y'_{i,s'} = \begin{cases} \frac{(Y+ZA)B_i}{1+ZB} - A_i + \frac{(Y_s-Y)B_i}{(1+ZB)} & \beta < \hat{\beta} \\ \frac{(Y+ZA)B_i}{1+ZB} - A_i + \frac{(Y_s-Y)\hat{R}_i}{(1+Z\hat{R})} & \beta \geq \hat{\beta} \end{cases}$$

We can also show that equation $y'_{i,s'} = 0$ only holds at $\beta = \hat{\beta}$. In addition, $y'_{i,s'}$ is a continuous function. These mean $y'_{i,s'}$ is either entirely positive or entirely negative in each of $[0, \hat{\beta}]$ or $[\hat{\beta}, \infty)$. Firstly, we prove that it is positive in $[0, \hat{\beta}]$.

We see that $\frac{dy'_{i,s'}}{dA_i} < 0$. On the other hand,

$$\frac{dA_i}{d\beta} = \frac{(Y-\alpha)\left(1+Z(n-1)B_i\right)^2(1+ZnB_i)}{\left(Z(n+1)+\delta+Z(n-1)(Zn+\beta)B_i\right)^2} \geq 0$$

Therefore, for $\beta < \hat{\beta}$, $\frac{dy'_{i,s'}}{d\beta} = \frac{dy'_{i,s'}}{dA_i}\frac{dA_i}{d\beta}$ is not positive. It means $y_{i,s'}$ is a non-increasing function of β in this interval. Considering the fact that $y'_{i,s'}(\hat{\beta}) = 0$, we can conclude

$$y'_{i,s'} \geq 0 \;\text{ if }\; \beta \leq \hat{\beta}. \tag{35}$$

Right derivative of $y'_{i,s'}$ at $\hat{\beta}$ also has a positive value of $\frac{Z^2(Y-Y_s)B_i(n-1)(1+Z(n-1)B_i)(1+ZnB_i)^2}{\sqrt{Z^2(n-2)^2+2Zn\beta+\delta^2}(Z(n+1)+\delta+ZB_i(-\beta+2n(Zn+\beta)+Z(n-1)n(Zn+\beta)B_i))^2}$.

If we add this to the facts that $y'_{i,s'}(\hat{T}) = 0$ and $y'_{i,s'}$ is either entirely non-negative or entirely non-positive for $\beta > \hat{\beta}$, we can conclude that

$$y'_{i,s'} \geq 0 \,\text{if}\, \beta \geq \hat{\beta} \tag{36}$$

(35) and (36) can be gathered to conclude

$$y'_{i,s'} \geq 0.$$

Therefore,

$$y_{i,s} = q_i^* + x_{i,s}^* \geq 0$$

We know from Lemma 2 that $x_{i,s}^*$ is non-positive for at least one-scenario. Thus,

$$q_i^* \geq 0$$

$\square$

6 Equilibrium of the stochastic settlement mechanism with non-negativity constraints: Theorem 3

6.1 SP clearing problem with non-negativity constraints

The SP clearing problem with non-negativity constraints is

ISOSP :

$$\begin{aligned}
\min\, z \quad &= \quad \sum_{s=1}^{S} \theta_s \left(\sum_{i=1}^{n} \left[a_i(q_i + x_{i,s}) + \frac{b}{2}(q_i + x_{i,s})^2 + \frac{d_i}{2} x_{i,s}^2 \right] - (Y_s C_s - \frac{Z}{2} C_s^2) \right) \\
s.t. \quad & \quad \sum_i q_i - Q = 0 \\
& \quad Q + \sum_i x_{i,s} - C_s = 0 \quad \forall s \in \{1, \ldots, S\} \\
& \quad q_i + x_{i,s} \geq 0 \quad \forall i, s \in \{1, \ldots, S\}
\end{aligned}$$

ISOSP is a convex optimization problem as the objective function of ISOSP is a convex function, and its constraints are linear. Therefore, solving the KKT conditions of this problem is equivalent to solving ISOSP.

6.1.1 KKT of ISOSP

To find the KKT conditions we can use the Lagrangian function

$$\begin{aligned}
L = \sum_{s=1}^{S} \Bigg(\theta_s \Bigg(\sum_{i=1}^{n} \left(a_i \left(x_{i,s} + q_i \right) + \frac{b_i}{2} \left(x_{i,s} + q_i \right)^2 + \frac{d_i}{2} x_{i,s}^2 \right) \\
- \left(C_s Y_s - \frac{Z C_s^2}{2} \right) + p_s \left(Q + \sum_{i=1}^{n} x_{i,s} - C_s \right) \Bigg) \\
- \sum_{i=1}^{n} e_{i,s} \left(x_{i,s} + q_i \right) \Bigg) - f \left(\sum_{i=1}^{n} q_i - Q \right).
\end{aligned}$$

To produce the building blocks of the KKT condition, we can use the partial derivations of L with respect to the decision variables.

$$\frac{dL}{dq_i} = -f - \sum_{s=1}^{S} e_{i,s} + (a_i + bq_i) + b\sum_{s=1}^{S}\theta_s x_{i,s}$$
$$\frac{dL}{dx_{i,s}} = -e_{i,s} + \theta_s\left(-p_s + a_i + b_i q_i + (b_i + d_i)x_{i,s}\right)$$
$$\frac{dL}{dC_s} = \left(p_s + ZC_s - Y_s\right)\theta_s$$
$$\frac{dL}{dQ} = f - \sum_{s}\theta_s p_s$$
$$\frac{dL}{dp_s} = \theta_s\left(C_s - \left(Q + \sum_{i=1}^{n} x_{i,s}\right)\right)$$
$$\frac{dL}{df} = Q - \sum_{i=1}^{n} q_i$$
$$\frac{dL}{de_{i,s}} = -q_i - x_{i,s}$$

Thus, KKT of this problem can be represented as

$$-f - \sum_{s=1}^{S} e_{i,s} + (a_i + b_i q_i) + b_i\sum_{s=1}^{S}\theta_s x_{i,s} = 0 \qquad \forall i \in \{1,\ldots,n\} \qquad \text{[C1]}$$
$$Q = \sum_{i=1}^{n} q_i \qquad \text{[C2]}$$
$$C_s = \left(Q + \sum_{i=1}^{n} x_{i,s}\right) \qquad \forall s \in \{1,\ldots,S\} \qquad \text{[C3]}$$
$$p_s = (Y_s - ZC_s) \qquad \forall s \in \{1,\ldots,S\} \qquad \text{[C4]}$$
$$f = \sum_{s=1}^{S}\theta_s p_s \qquad \text{[C5]}$$
$$e_{i,s} = \theta_s\left(-p_s + a_i + b_i q_i + (b_i + d_i)x_{i,s}\right) \qquad \forall i \in \{1,\ldots,n\}\ \forall s \in \{1,\ldots,S\} \qquad \text{[C6]}$$
$$e_{i,s}(q_i + x_{i,s}) = 0 \qquad \forall i \in \{1,\ldots,n\}\ \forall s \in \{1,\ldots,S\} \qquad \text{[C7]}$$
$$e_{i,s} \geq 0 \qquad \forall i \in \{1,\ldots,n\}\ \forall s \in \{1,\ldots,S\} \qquad \text{[C8]}$$
$$q_i + x_{i,s} \geq 0 \qquad \forall i \in \{1,\ldots,n\}\ \forall s \in \{1,\ldots,S\}. \qquad \text{[C9]}$$

If we replace the value of f and $e_{i,s}$ from [C5] and [C6] into [C1], constraint [C1] can be replaced with $\sum_{s=1}^{S}\theta_s x_{i,s} = 0$.

6.1.2 Firms' optimisation problem

Problem WNN[j] represents the optimization problem solved by firm j to maximize its profit, subject to KKT conditions of ISO's optimization problem.

WNN[j]:

$$\max\ u_j = \sum_{s=1}^{S} \theta_s \Bigg(p_s(q_j + x_{j,s}) - \\ \left(\alpha_j \left(q_j + x_{j,s}\right) + \frac{\beta_j}{2} \left(q_j + x_{j,s}\right)^2 + \frac{\delta_j}{2} {x_{j,s}}^2 \right) \Bigg)$$

$$\text{s.t.} \quad \sum_{s=1}^{S} \theta_s x_{i,s} = 0 \qquad \forall i \in \{1, \ldots, n\} \quad [\text{C1}]$$

$$Q = \sum_{i=1}^{n} q_i \qquad [\text{C2}]$$

$$C_s = \left(Q + \sum_{i=1}^{n} x_{i,s} \right) \qquad \forall s \in \{1, \ldots, S\} \quad [\text{C3}]$$

$$p_s = (Y_s - ZC_s) \qquad \forall s \in \{1, \ldots, S\} \quad [\text{C4}]$$

$$f = \sum_{s=1}^{S} \theta_s p_s \qquad [\text{C5}]$$

$$e_{i,s} = \theta_s \left(-p_s + a_i + b_i q_i + (b_i + d_i) x_{i,s} \right) \qquad \forall i \in \{1, \ldots, n\},\ \forall s \in \{1, \ldots, S\} \quad [\text{C6}]$$

$$e_{i,s}(q_i + x_{i,s}) = 0 \qquad \forall i \in \{1, \ldots, n\},\ \forall s \in \{1, \ldots, S\} \quad [\text{C7}]$$

$$e_{i,s} \geq 0 \qquad \forall i \in \{1, \ldots, n\},\ \forall s \in \{1, \ldots, S\} \quad [\text{C8}]$$

$$q_i + x_{i,s} \geq 0 \qquad \forall i \in \{1, \ldots, n\},\ \forall s \in \{1, \ldots, S\} \quad [\text{C9}]$$

To make the optimization problem look simpler, we can replace the values of Q, C_s, and f from [C2], [C3], and [C5] in the other equations. This simplifies

WNN to the following shape.

WNN[j]:

$$\max\ u_j = \sum_{s=1}^{S} \theta_s \Bigg(p_s(q_j + x_{j,s}) - \left(\alpha_j\,(q_j + x_{j,s}) + \frac{\beta_j}{2}\ (q_j + x_{j,s})^{\,2} + \frac{\delta_j}{2} {x_{j,s}}^2 \right) \Bigg)$$

$$\begin{aligned}
\text{s.t.}\quad & \sum_{s=1}^{S} \theta_s x_{i,s} = 0 && \forall i \in \{1,\dots,n\} && \text{[C1]}\\
& p_s = Y_s - Z\left(\sum_{h=1}^{n} q_h + \sum_{h=1}^{n} x_{h,s} \right) && \forall s \in \{1,\dots,S\} && \text{[C4]}\\
& e_{i,s} = -\theta_s\,(-p_s + a_i + b_i q_i + (b + d_i) x_{i,s}) && \forall i \in \{1,\dots,n\} && \text{[C6]}\\
& && \forall s \in \{1,\dots,S\} &&\\
& e_{i,s}(q_i + x_{i,s}) = 0 && \forall i \in \{1,\dots,n\} && \text{[C7]}\\
& && \forall s \in \{1,\dots,S\} &&\\
& e_{i,s} \geq 0 && \forall i \in \{1,\dots,n\} && \text{[C8]}\\
& && \forall s \in \{1,\dots,S\} &&\\
& q_i + x_{i,s} \geq 0 && \forall i \in \{1,\dots,n\} && \text{[C9]}\\
& && \forall s \in \{1,\dots,S\} &&
\end{aligned}$$

With a similar process, the optimization problem of firm j in a stochastic market clearing mechanism without non-negativity constraints can be found as

WONN[j]:

$$\max\ u_j = \sum_{s=1}^{S} \theta_s \Bigg(p_s(q_j + x_{j,s}) - \left(\alpha_j\,(q_j + x_{j,s}) + \frac{\beta_j}{2}\ (q_j + x_{j,s})^{\,2} + \frac{\delta_j}{2} {x_{j,s}}^2 \right) \Bigg)$$

$$\begin{aligned}
\text{s.t.}\quad & \sum_{s=1}^{S} \theta_s x_{i,s} = 0 && \forall i \in \{1,\dots,n\} && \text{[C1]}\\
& p_s = Y_s - Z\left(\sum_{h=1}^{n} q_h + \sum_{h=1}^{n} x_{h,s} \right) && \forall s \in \{1,\dots,S\} && \text{[C4]}\\
& e_{i,s} = \theta_s\,(-p_s + a_i + b_i q_i + (b_i + d_i) x_{i,s}) && \forall i \in \{1,\dots,n\} && \text{[C6]}\\
& e_{i,s} = 0 && \forall i \in \{1,\dots,n\} && \text{[C11]}\\
& && \forall s \in \{1,\dots,S\}. &&
\end{aligned}$$

Also, we introduce a relaxation to WNN, which we use later in proofs of our theorems. We eliminate constraint [C7]: $e_{i,s}(q_i + x_{i,s}) = 0$, and limit the constraint [C9]: $\forall i, q_i + x_{i,s} \geq 0$ to the optimizer generator j to obtain a relaxation problem

RWNN:

$$\max\ u_j = \sum_{s=1}^{S} \theta_s \Bigg(p_s(q_j + x_{j,s}) - \\ \left(\alpha_j \left(q_j + x_{j,s}\right) + \frac{\beta_j}{2}\ \left(q_j + x_{j,s}\right)^2 + \frac{\delta_j}{2} {x_{j,s}}^2 \right) \Bigg)$$

$$\begin{aligned}
\text{s.t.}\quad & \sum_{s=1}^{S} \theta_s x_{i,s} = 0 && \forall i \in \{1,\dots,n\} && \text{[C1]}\\
& p_s = Y_s - Z\left(\sum_{h=1}^{n} q_h + \sum_{h=1}^{n} x_{h,s} \right) && \forall s \in \{1,\dots,S\} && \text{[C4]}\\
& f = \sum_{s=1}^{S} \theta_s p_s && && \text{[C5]}\\
& e_{i,s} = \theta_s \left(-p_s + a_i + b_i q_i + (b_i + d_i) x_{i,s}\right) && \forall i \in \{1,\dots,n\} && \text{[C6]}\\
& && \forall s \in \{1,\dots,S\} && \\
& e_{i,s} \geq 0 && \forall i \in \{1,\dots,n\} && \text{[C8]}\\
& q_j + x_{j,s} \geq 0 && \forall s \in \{1,\dots,S\}. && \text{[C12]}
\end{aligned}$$

Now, we prove three lemmas which help us to demonstrate the final theorem.

Lemma 6.1. *If for every $i \neq j$ (j is the optimizer generator), a_i and b_i has the same value, then the constraint $e_{i,s} \geq 0$ (for every $i \neq j$) in RWNN can be replaced with $e_{i,s} = 0$ without reducing the optimal value of RWNN.*

Proof. We prove the lemma by contradiction. Assume there exist a point $\nu = (a_j, b_j, \boldsymbol{q}, \boldsymbol{x}, \boldsymbol{p}, \boldsymbol{e})$ with at least one $e_{i',s'} > 0$ ($i \neq j$) and higher objective value than any feasible solution with $\boldsymbol{e} = \boldsymbol{0}$.

Consider $\nu' = (a'_j, b'_j, \boldsymbol{q}', \boldsymbol{x}', \boldsymbol{p}', \boldsymbol{e}')$ defined as follows.

$$q'_i = \begin{cases} q_i & i = j \\ q_i + \frac{\frac{Z\sum_{h\neq j}\sum_w e_{h,w}}{Z(n-1)+b_i} - \sum_w e_{i,w}}{b_i} & i \neq j \end{cases} \tag{37}$$

$$x'_{i,s} = \begin{cases} x_{i,s} & i = j \\ x_{i,s} + \frac{\sum_w e_{i,w} - \frac{e_{i,s}}{\theta_s}}{b_i+d_i} - \frac{Z(\sum_{h\neq j}\sum_w e_{h,w} - \sum_{h\neq j}\frac{e_{h,s}}{\theta_s})}{(Z(n-1)+b_i+d_i)(b_i+d_i)} & i \neq j \end{cases} \tag{38}$$

$$a'_j \geq \max_s \left\{ Z\left(\sum_{h\neq j}\sum_w e_{h,w}\left(\frac{1}{Z(n-1)+b_i} - \frac{1}{Z(n-1)+b_i+d_i}\right) + \frac{\sum_{h\neq j}\frac{e_{h,s}}{\theta_s}}{Z(n-1)+b_i+d_i}\right) + a_j \right\} \tag{39}$$

$$d'_j = d_j \tag{40}$$

Firstly, we show this is a feasible solution.

$$\sum_s \theta_s x'_{i,s} = \begin{cases} \sum_s \theta_s x_{i,s} & i = j \\ \sum_s \theta_s x_{i,s} + \frac{\sum_w e_{i,w} - \sum_s \theta_s \frac{e_{i,s}}{\theta_s}}{b_i+d_i} - \frac{Z(\sum_{h\neq j}\sum_w e_{h,w} - \sum_s \theta_s \sum_{h\neq j}\frac{e_{h,s}}{\theta_s})}{(Z(n-1)+b_i+d_i)(b_i+d_i)} & i \neq j \end{cases}$$

Extra simplifications yields to

$$\forall i : \sum_s \theta_s x'_{i,s} = 0 \tag{41}$$

After substituting the value of q'_h from (37) into $\sum_{h\neq j} q'_h$ and slightly simplifying the resulted equation, we get

$$\sum_{h\neq j} q'_h = \sum_{h\neq j} q_h - \frac{\sum_{h\neq j}\sum_w e_{h,w}}{Z(n-1)+b_i} \tag{42}$$

The same analysis on equation (38) gives us the following equation.

$$\sum_{h\neq j} x'_{h,s} = \sum_{h\neq j} x_{h,s} + \frac{\sum_{h\neq j}\sum_w e_{h,w} - \sum_{h\neq j}\frac{e_{h,s}}{\theta_s}}{Z(n-1)+b_i+d_i} \tag{43}$$

p'_s can be obtained combining equations [C4], (42), and (43).

$$\begin{aligned} p'_s =& p_s - Z\left(-\frac{\sum_{h\neq j}\sum_w e_{h,w}}{Z(n-1)+b_i} + \frac{\sum_{h\neq j}\sum_w e_{h,w} - \sum_{h\neq j}\frac{e_{h,s}}{\theta_s}}{Z(n-1)+b_i+d_i}\right) \\ =& p_s + Z\left(\sum_{h\neq j}\sum_w e_{h,w}\left(\frac{1}{Z(n-1)+b_i} - \frac{1}{Z(n-1)+b_i+d_i}\right) \right. \\ &\left. + \frac{\sum_{h\neq j}\frac{e_{h,s}}{\theta_s}}{Z(n-1)+b_i+d_i}\right) \end{aligned} \tag{44}$$

Considering the fact that $e_{i,s}$, Z, b_i, and d_i have non-negative values,

$$p'_s \geq p_s \tag{45}$$

From (37), (38), (44), and [C6], $e_{i,s}$ can be obtained as follows.

$$e'_{i,s} = \begin{cases} e_{i,s} + \theta_s(-p'_s + p_s + a'_j - a_j) & i = j \\ e_{i,s} + \theta_s \Bigg(- Z \sum_{h \neq j} \sum_w e_{h,w} \left(\frac{1}{Z(n-1)+b_i} - \frac{1}{Z(n-1)+b_i+d_i} \right) & i \neq j \\ \quad -Z \frac{\sum_{h \neq j} \frac{e_{h,s}}{\theta_s}}{Z(n-1)+b_i+d_i} + \frac{Z \sum_{h \neq j} \sum_w e_{h,w}}{Z(n-1)+b_i} - \sum_w e_{i,w} & \\ \quad + \sum_w e_{i,w} - \frac{e_{i,s}}{\theta_s} - \frac{Z(\sum_{h \neq j} \sum_w e_{h,w} - \sum_{h \neq j} \frac{e_{h,s}}{\theta_s})}{Z(n-1)+b_i+d_i} \Bigg) & \end{cases}$$

This simplifies to

$$e'_{i,s} = \begin{cases} e'_{j,s} \geq 0 & i = j \\ 0 & i \neq j \end{cases}$$

Thus, the constraint [C8] is also satisfied. As $q'_j = q_j$, $x'_{j,s} = x_{j,s}$, and ν is a feasible solution, constraints [C12] are also fulfilled.

In sum, ν' is a feasible solution.

On the other hand, a comparison between the u'_j and u_j demonstrates that ν' gives a better objective:

$$u'_j - u_j = \sum_s \theta_s (p'_s - p_s)(q_j + x_{j,s}).$$

With $q_j + x_{j,s} \geq 0$, as concluded from [C12], and $p'_s - p_s \geq 0$ as resolved in (45)

$$u'_j \geq u_j$$

This contradicts the initial assumption, which proves the lemma. □

Lemma 6.2. *RWNN can be simplified to the following optimization problem.*

RWNN:

$$
\begin{aligned}
\max\ u_j = {} & f q_j + \sum_{s=1}^{S} \theta_s (p_s - f) x_{j,s} \\
& - \left(\alpha_j q_j + \frac{\beta_j}{2} q_j^2 + \frac{\beta_j + \delta_j}{2} \sum_{s=1}^{S} \theta_s {x_{j,s}}^2 \right)
\end{aligned}
$$

$$
\begin{array}{lllr}
s.t. & \displaystyle\sum_{s=1}^{S} \theta_s x_{i,s} = 0 & \forall i \in \{1,\ldots,n\} & \textit{[C1]} \\
& p_s = Y_s - Z \left(\displaystyle\sum_{h=1}^{n} q_h + \sum_{h=1}^{n} x_{h,s} \right) & \forall s \in \{1,\ldots,S\} & \textit{[C4]} \\
& f = \displaystyle\sum_{s=1}^{S} \theta_s p_s & & \textit{[C5]} \\
& e_{i,s} = \theta_s \left(-p_s + a_i + b_i q_i + (b_i + d_i) x_{i,s} \right) & \forall i \in \{1,\ldots,n\} & \textit{[C6]} \\
& & \forall s \in \{1,\ldots,S\} & \\
& e_{i,s} \geq 0 & \forall i \in \{1,\ldots,n\} & \textit{[C8]} \\
& q_j + x_{j,s} \geq 0 & \forall s \in \{1,\ldots,S\} & \textit{[C12]}
\end{array}
$$

Proof. The first part of the objective function is the optimizer's income, which is equal to

$$
\begin{aligned}
\sum_{s=1}^{S} \theta_s p_s (q_j + x_{j,s}) = & \sum_{s=1}^{S} \theta_s p_s q_j + \sum_{s=1}^{S} \theta_s p_s x_{j,s} \\
= & f q_j + \sum_{s=1}^{S} \theta_s f x_{j,s} + \sum_{s=1}^{S} \theta_s (p_s - f) x_{j,s} && \text{From [C5]} \\
= & f q_j + \sum_{s=1}^{S} \theta_s (p_s - f) x_{j,s} + f \sum_{s=1}^{S} \theta_s x_{j,s} \\
= & f q_j + \sum_{s=1}^{S} \theta_s (p_s - f) x_{j,s} && \text{From [C1]}
\end{aligned}
$$

The rest of the objective function can also be simplified similarly, as follows.

$$\begin{aligned}
\text{Generating Cost} = &\sum_{s=1}^{S} \theta_s \left(\alpha_j \left(q_j + x_{j,s}\right) + \frac{\beta_j}{2} \ \left(q_j + x_{j,s}\right)^2 + \frac{\delta_j}{2} {x_{j,s}}^2 \right) \\
= &\alpha_j q_j + \frac{\beta_j}{2} q_j^2 + \frac{\beta_j + \delta_j}{2} \sum_{s=1}^{S} \theta_s {x_{j,s}}^2 \\
&+ \left(\alpha_j + \beta_j q_j\right) \sum_{s=1}^{S} \theta_s x_{j,s} \\
= &\alpha_j q_j + \frac{\beta_j}{2} q_j^2 + \frac{\beta_j + \delta_j}{2} \sum_{s=1}^{S} \theta_s {x_{j,s}}^2 && \text{From [C1]}
\end{aligned}$$

□

Lemma 6.3. *If for every $i \neq j$ (j is the optimizer generator), a_i and d_i has the same value, then the optimal solution to WONN is at least as good as the optimal value to RWNN.*

Proof. To prove the lemma, we find the optimal solution to RWNN, while we ignore the non-negativity constraint $q_j + x_{j,s} \geq 0$. Thus, this point gives an objective value as good as (possibly better than) the optimal point. Then we show this point is a feasible solution to WONN, which proves the lemma.

From lemma 6.2 we have

$$e_{i,s} = \theta_s \left(-Y_s + Z \left(\sum_{h=1}^{n} q_h + \sum_{h=1}^{n} x_{h,s} \right) + a_i + b_i q_i + (b_i + d_i) x_{i,s} \right).$$

To simplify the equations we use some transformations. Let $R_i = \frac{1}{(b_i + d_i)}$, and $A_i = \frac{a_i}{b_i}$. Also, let A and R denote $\sum_{h=1}^{n} A_h$, and $\sum_{h=1}^{n} R_h$ respectively. Then, constraint [C6] looks like

$$e_{i,s} = \theta_s \left(-Y_s + Z \left(\sum_{h=1}^{n} q_h + \sum_{h=1}^{n} x_{h,s} \right) + \frac{1}{R_i} x_{i,s} + b_i \left(A_i + q_i\right) \right). \tag{46}$$

A summation over different scenarios gives

$$\sum_{w=1}^{S} e_{i,w} = -Y + Z \sum_{h=1}^{n} q_i + \left(A_i + q_i\right) b_i \tag{47}$$

From lemma 6.1, the constraints $e_{i,s} = 0$ for every $i \neq j$ and s can be replaced with $e_{i,s} \geq 0$ in RWNN. On the other hand, from the assumption we know that A_i has a fixed value for every $i \neq j$. As a result, equation (47) is used to show that q_i must have a fixed value for every $i \neq j$. Thus, equation (47) can be re-written as

$$0 = -Y + Z\left((n-1)q_i + q_j\right) + \left(A_i + q_i\right)b_i \tag{48}$$

With a similar argument, we can show that $x_{i,s}$ also has the same value for every $i \neq j$. Equation (46), thus, can be represented as

$$0 = \theta_s \left(-Y_s + Z\left((n-1)q_i + q_j + (n-1)x_{i,s} + x_{j,s}\right) + \frac{1}{R_i}x_{i,s} + b_i\left(A_i + q_i\right)\right) \tag{49}$$

Solving equations (48) and (49), we find the values of q_i and $x_{i,s}$ as functions of q_j and $x_{j,s}$.

$$\begin{aligned} q_i =& \frac{Y - b_i A_i - Z q_j}{b_i + (n-1)Z} \\ x_{i,s} =& -\frac{R_i\left(Y - Y_s + Z x_{j,s}\right)}{1 + (n-1)ZR_i} \end{aligned} \tag{50}$$

From (50) we can also calculate the values of f and $p_s - f$ as functions of q_j and $x_{j,s}$.

$$\begin{aligned} f =& \frac{b_i\left(Y + (n-1)ZA_i - Zq_j\right)}{b_i + (n-1)Z} \\ p_s - f =& \frac{-Y + Y_s - Zx_{j,s}}{1 + (n-1)ZR_i} \end{aligned} \tag{51}$$

Inserting these values into the utility function from lemma 6.2 simplifies the utility function to

$$\begin{aligned} u_j =& \left(\frac{b_i\left(Y + (n-1)ZA_i - Zq_j\right)}{b_i + (n-1)Z} - \alpha_j - \frac{\beta_j}{2}q_j\right)q_j \\ &+ \sum_{s=1}^{S}\theta_s\left(\frac{-Y + Y_s - Zx_{j,s}}{1 + (n-1)ZR_i} - \frac{\beta_j + \delta_j}{2}x_{j,s}\right)x_{j,s} \end{aligned}$$

As Z, α_j, β_j, and R_i have non-negative values, u_j is a concave function of q_j and $\boldsymbol{x}_j$. Therefore, ignoring the rest of the constraints, the optimal value of q_j and $x_{j,s}$ can be found using first order conditions.

First order conditions for q_j and $x_{j,s}$ gives

$$q_j^* = \frac{b_i Y + (n-1) b_i Z A_i - (b_i + (n-1)Z)\alpha_j}{2 b_i Z + (b_i + (n-1)Z)\beta_j} \tag{52}$$

$$x_{j,s}^* = \frac{Y_s - Y}{2Z + (1 + (n-1)Z R_i)(\beta_j + \delta_j)}. \tag{53}$$

Now we need to show that we can always find A_j and R_j, so that this value is a feasible solution to WONN and yields $e_{j,s} = 0$. To do so, we first calculate $\frac{e_{j,s}}{\theta_s} - \sum_w e_{j,w}$ for all s. From (46), (47), and (52)

$$\begin{aligned}\frac{e_{j,s}}{\theta_s} - \sum_{w=1}^{S} e_{j,w} =& Y - Y_s + \frac{x_{j,s}}{R_j} + Z\left((n-1)x_{i,s} + x_{j,s}\right)\\ =& \frac{(Y - Y_s)\left(-1 + R_j\left(Z + \beta_j + \delta_j\right) + (n-1) Z R_i \left(-1 + R_j\left(\beta_j + \delta_j\right)\right)\right)}{R_j\left(1 + (n-1)Z R_i\right)\left(2Z + \left(1 + (n-1) Z R_i\right)\left(\beta_j + \delta_j\right)\right)}\end{aligned} \tag{54}$$

It is always possible to choose R_j as follows to ensure that $\frac{e_{j,s}}{\theta_s} - \sum_w e_{j,w} = 0$. Note that this does not change either of production quantities or prices. This value of R_j is

$$R_j = \frac{1 + (n-1) Z R_i}{Z + \left(1 + (n-1) Z R_i\right)\left(\beta_j + \delta_j\right)}$$

We can also choose A_j so that $\sum_w e_{j,w} = 0$ without changing any production quantity and thus any prices. From (47) and (50)

$$\begin{aligned}\sum_{w=1}^{S} e_{j,w} =& -Y + Z\left((n-1)q_i + q_j\right) + b_i\left(A_h + q_h\right)\\ =& -Y + b_i\left(A_j + q_j\right) + \frac{(n-1)Z\left(Y - b_i A_i\right) + b_i Z q_j}{b_i + (n-1)Z}\end{aligned} \tag{55}$$

Solving $\sum_w e_{j,w} = 0$ for A_j gives

$$\begin{aligned}A_j =& \frac{-b_i Y (b_i + (n-2)Z) + (b_i + (n-1)Z)\left((b_i + nZ)\alpha_j + Y\beta_j\right)}{(b_i + (n-1)Z)\left(2 b_i Z + (b_i + (n-1)Z)\beta_j\right)}\\ &+ \frac{-(n-1) Z A_i \left(b_i (b_i + (n-2)Z) - (b_i + (n-1)Z)\beta_j\right)}{(b_i + (n-1)Z)\left(2 b_i Z + (b_i + (n-1)Z)\beta_j\right)}\end{aligned}$$

These A_j and R_j ensures $\begin{aligned}\sum_{w=1}^{S} e_{j,w} =& 0\\ \frac{e_{j,s}}{\theta_s} - \sum_{w=1}^{S} e_{j,w} =& 0\end{aligned} \Rightarrow \forall s : e_{j,s} = 0$

Thus, constraints [C6] and [C8] are met in WONN and RWNN.

From (52) we derive $\sum_s \theta_s x_{j,s} = 0$. We can use the fact that $\sum_s \theta_s x_{j,s} = 0$ to show that for $i \neq j$ also $\sum_s \theta_s x_{i,s} = 0$ (in equation (50)). So, this optimal point is feasible in [C1].

In sum, the constructed point is feasible to WONN, and gives an objective value at least as good as RWNN. □

Now, we can use the above lemmas to prove a theorem that shows using the equilibrium of the simplifies game without the non-negativity constraints instead of the equilibrium of the original game is justifiable.

Theorem. *The equilibrium of the symmetric SFSP game without the non-negativity constraints in ISO's problem is also the equilibrium of SFSP game with the non-negativity constraints.*

Proof. To prove the theorem, we should show that if all generators offer the equilibrium values of a_i and d_j none of them are willing to deviate from it. Equivalently, if in WNN a_i and d_i are equal to the equilibrium of the SFSP game without the non-negativity constraints for all $i \neq j$, then optimal a_j and d_j are also equal to equilibrium values of this game.

The equilibrium of SFSP without non-negativity constraints is equal to the optimal value of WONN when every non-optimizer generator has offered the equilibrium values of the game. Thus, we prove that the optimal value of WONN is also optimal to WNN.

Firstly, lemma 6.3 states that if the optimal solution to WONN is feasible to RWNN, then, it is also the optimal solution to RWNN. In our problem, from theorem 4.9 we know that the optimal solution to WONN holds both $q_i \geq 0$ and $q_i + x_{i,s} \geq 0$. The other constraints of RWNN are shared between these two models. Thus, it is feasible and optimal in RWNN.

On the other hand, every feasible solution to WNN is feasible in RWNN. So, if this solution (which is the optimal solution to RWNN) is feasible to WNN, then it is also optimal to WNN. From the theorem 4.9, we know that $q_i \geq 0$ and $q_i + x_{i,s} \geq 0$ for all i, as it is the equilibrium of the game without non-negativity constraints. This means this point is feasible in [C8] and [C9]. On the other hand, we know that $e_{i,s} = 0$ for all i, as it is the optimal solution to WONN. This shows it also holds [C7]. The other constraints are common and thus met. In sum, This point is feasible and therefore optimal to WNN.

Thus, no generator is willing to deviate from this point unilaterally, and this is the equilibrium of WNN. □

7 Computations for the case with zero intercept (similar to Green 1996)

The following is the outline of the computations for comparison of NZTS and ISOSP but using linear supply functions with intercept zero (as per Green). We have the same linear demand function and generator cost structure as before.

This time however the generators are restricted to offers of the form $p = \frac{1}{\alpha}q$ for firm i. Following the notation of Green, we allow for demand curves of the form $D(p) = D^s - \gamma p$, where D^s is the realization of demand in scenario s. Throughout this computation, we assume symmetry and we will assume a duopoly.

Note: There is a correspondence between our notation here and that used in the main body of the paper for general affine supply functions, however, transformations of the kind used in the main paper make the computations in this case cumbersome. We will state the correspondence here and return to the notation used by Green for this special case.

Green's notation	notation used for the affine case
α	$\frac{1}{b_i}$
γ	$\frac{1}{Z}$
D	$\frac{Y}{Z}$
σ_D^2	$\frac{\sigma_Y^2}{Z^2}$

For NZTS,

We set up the market clearing problems for the two periods of NZTS and derive that

$$p_s = \frac{D^s}{2\alpha + \gamma} \tag{56}$$

$$y_{i,s} = \frac{\alpha D^s}{2\alpha + \gamma} \tag{57}$$

$$q_i = \frac{D\alpha}{2\alpha + \gamma} \tag{58}$$

$$f = \frac{D}{2\alpha + \gamma}. \tag{59}$$

Knowing the results of market clearing, a firm can proceed to optimize their profit, given the other firm's bids. We assume the firm's fixed cost here is zero. This renders the following optimization problem.

$$\max_{\alpha} E[p_s y_{i,s} - \frac{\beta}{2} - \frac{\delta}{2}(y_{i,s} - q_i)^2]$$

Using the expressions in (56) and substituting into the profit maximization problem above, subsequently taking the expectation, we obtain

$$\max_{\alpha} \frac{1}{2(2\alpha+\gamma)^2}(\alpha^2(-\beta\sigma_D^2 - \beta D - \delta\sigma_D^2) + 2(\sigma_D^2 + D^2)\alpha)$$

Here σ_D^2 is the variance of D^s. Using Mathematica (notebooks available upon request), we obtain that the objective is quasi-concave with a maximizer located at

$$\alpha = \frac{(\alpha_- + \gamma)(D^2 + \sigma_D^2)}{(1 + \beta(\alpha_- + \gamma))D^2 + (1 + \beta(\alpha_- + \gamma) + \alpha_-\delta + \gamma\delta)\sigma_D^2}$$

where α_- is the given slope of the opponent's supply offer. Of course the analogous expression for α_- will provide the maximizer for the opponent. Solving these together yields the equilibrium values of the respective slopes. In particular, under the assumption of symmetry

$$\alpha = \frac{\sqrt{\gamma\left((4+\beta\gamma)D^2+(4+\beta\gamma+\gamma\delta)\sigma_D^2\right)\left(\delta\sigma_D^2+\beta(D^2+\sigma_D^2)\right)}-\beta\gamma(D^2+\sigma_D^2)-\gamma\delta\sigma_D^2}{2(\delta\sigma_D^2+\beta(D^2+\sigma_D^2)}$$

The quasi-concavity is proved by observing that the derivative of the profit function is increasing to the left of the critical α and decreasing to the right of it.

We can substitute this α into the (56), and then substitute these expressions in the total welfare for NZTS below.

$$TWNZTS = \frac{1-\gamma H}{2}\left(1-\frac{\gamma(1-\gamma H)}{2}\right)(\sigma_D^2+D^2) - \frac{c+\delta}{8}(1-\gamma H)^2(2\sigma_D^2+D^2)$$

where

$$H = \frac{\beta D^2+(\beta+\delta)\sigma_D^2}{\sqrt{\gamma(\beta D^2+(\beta+\delta)\sigma_D^2)((4+\beta\gamma)D^2+(4+\gamma(\beta+\delta))\sigma_D^2)}}$$

.

For ISOSP, In this section we return to the notation used in the main body of the paper as the transformations used there would assist the computations here.

Here the stochastic programming market clearing mechanism separates into 2 terms, just like that of the main body of the paper (the only difference is that the supply function intercepts are 0 and this does not change anything). We derive that

$$q_i = \frac{YB_i}{1+ZB} \tag{60}$$

$$f = \frac{Y}{1+ZB}. \tag{61}$$

$$x_{i,s} = \frac{(Y^s - Y)R_i}{1+ZR} \tag{62}$$

$$p_s = \frac{Y}{1+ZB} + \frac{(Y^s - Y)}{(1+ZB)} \tag{63}$$

Recall here that $B_i = \frac{1}{b_i} = \alpha$ (in the notation of Green) and $R_i = \frac{1}{d_i}$ where d_i is the deviation penalty bid in. Similar to the above, we assume symmetry and duopoly throughout computing the equilibrium here.

Using the expressions in (60) and substituting into the expression for the firm's expected profit, we obtain the objective in the profit maximization problem below

$$max_{B_i,R_i} u^i = \frac{2B_iY^2 - B_i^2Y^2\beta}{2(1+ZB_i+ZB_{-i})^2} + \frac{(-2R_i + R_i^2(\beta+\delta))(Y^2 - E[Y_s^2])}{2(1+ZR_i+ZR_{-i})^2}$$

Note that the objective function (expected utility of firm i), decouples in B_i and R_i. Therefore very similar to the main body of the paper, we establish quasi-concavity in each variable and obtain the maximizing values of B_i and R_i.

We used Mathematica to find the expressions for derivatives of u^i.

$$\frac{du^i}{dB_i} = -\frac{Y^2(-1 - ZB_{-i} + B_i(Z+\beta+Z\beta B_{-i}))}{(1+ZB_i+ZB_{-i})^3}$$

Note that the numerator is linear in B_i. The stationary point for B_i is then

$$B_i^* = \frac{1+ZB_{-i}}{Z+\beta+Z\beta B_{-i}}$$

This derivative increases for values of B_i to the left of B_i^* and decreases to the right. This establishes the quasi-concavity the first part.

Similarly we can compute $\frac{du^i}{dR_i}$, find the critical point and establish quasi-concavity in R_i. The critical point R_i^* is given by

$$R_i^* = \frac{1+ZR_{-i}}{Z+\beta+\delta+Z\beta R_{-i}+Z\delta R_{-i}}.$$

Now, u^i being separable in B_i and R_i is proved to be quasi-concave and the optimal solution to the profit maximization problem is given by B_i^*, R_i^*.

We can now use our assumption of symmetry (and duopoly) to solve for the equilibrium values and obtain:

$$B_i^E = \frac{2}{\beta + \sqrt{\beta(4Z+\beta)}},$$

$$R_i^E = \frac{\sqrt{4Z+\beta+\delta} - \sqrt{\beta+\delta}}{2Z\sqrt{\beta+\delta}}.$$

We can then compute the equilibrium welfare value. This expression which we refer to as $TW - ISOSP$ is given by

$$\frac{2Y^2}{4Z+\beta} + \frac{2\sigma_Y^2}{4Z+\beta+\delta} + \frac{4Y^2\beta}{(4Z+\beta+\sqrt{\beta(4Z+\beta)})^2}$$
$$+ \frac{\sigma_Y^2(\beta+\delta)(Z(4\sqrt{\beta+\delta} - 2\sqrt{4Z+\beta+\delta}) + (\beta+\delta)(\sqrt{\beta+\delta} - \sqrt{4Z+\beta+\delta})}{2Z^2(4Z+\beta+\delta)^{\frac{3}{2}}}.$$

In the Mathematica notebook (ISOSP-Eu-Eqm.nb), we have converted the expression above to use Green parameters and in the notebook (Comparison-welfare.nb) we have a comparison with the total welfare rendered from NZTS. One can obtain ranges which each of the welfare expressions are greater than the other; these are visualized using the *manipulate* command. This of course does not happen when we allow for an intercept in the supply functions.

Specifically, if we fix $\beta = 2.0$, $\delta = 4.0$, $D = 10$, $\sigma_D^2 = 1$, then

$$TWD = TWNZTS - TWISOSP = \begin{cases} -4.26892 & \text{if } \gamma = 2 \\ 1.54577 & \text{if } \gamma = 8 \end{cases}$$

In the notation used in the main body of the paper, this translates to:

$\gamma = 2$, (TWD = -4.26892)	$\gamma = 8$, (TWD = 1.54577)
$Y = 5$	$Y = 1.25$
$\sigma_Y^2 = 0.25$	$\sigma_Y^2 = 0.0625$
$Z = 0.5$	$Z = 0.125$

8 Computations of firms and equilibrium values for the SP mechanism

Best response curves

From propositions 3.1 and 3.2, we have

$$Q = \frac{EY\,cB - cA}{1 + Z\,cB}$$

$$q_{i\_} = \frac{(EY + Z\,cA)\,B_i}{(1 + Z\,cB)} - A_i$$

$$x_{i\_,s\_} = \frac{(Y_s - EY)\,R_i}{1 + Z\,cR}$$

$$y_{i\_,s\_} = q_i + x_{i,s}$$

$$f = \frac{EY + Z\,cA}{1 + Z\,cB}$$

$$p_{s\_} = f + \frac{Y_s - EY}{1 + Z\,cR}$$

$$\theta_S = 1 - \sum_{s=1}^{S-1} \theta_s$$

$$cA = A_i + A_{-i}$$

$$cB = B_i + B_{-i}$$

$$cR = R_i + R_{-i}$$

$$u_{i\_} = \text{Simplify}\left[f\,q_i + \sum_{s=1}^{S} \theta_s \left(p_s\,x_{i,s} - \left(\alpha\,(q_i + x_{i,s}) + \frac{\beta}{2}\,(q_i + x_{i,s})^2 + \frac{\delta}{2}\,x_{i,s}^2\right)\right)\right];$$

$$\text{welfare} = \sum_{s=1}^{S} \theta_s \left(Y_s\,(n\,y_{1,s}) - \frac{Z}{2}\,(n\,y_{1,s})^2 - n\left(\alpha\,y_{1,s} + \frac{\beta}{2}\,y_{1,s}^2 + \frac{\delta}{2}\,x_{1,s}^2\right)\right);$$

$$\text{FullSimplify}\left[D[u_i, R_i, A_i], \left\{EY == \sum_{s=1}^{S} \theta_s\,Y_s,\ \theta_S = 1 - \sum_{s=1}^{S-1} \theta_s\right\}\right]$$

0

$$\text{FullSimplify}\left[D[u_i, R_i, B_i], \left\{EY == \sum_{s=1}^{S} \theta_s\,Y_s,\ \theta_S = 1 - \sum_{s=1}^{S-1} \theta_s\right\}\right]$$

0

Therefore, $u_i(A_i, B_i, R_i) = g_i(A_i, B_i) + h_i(R_i)$.

$$\text{FullSimplify}[D[u_i, A_i, A_i]]$$

$$-\frac{(1 + Z\,B_{-i})\,(2\,Z + \beta + Z\,\beta\,B_{-i})}{(1 + Z\,(B_{-i} + B_i))^2}$$

```
FullSimplify[Solve[D[u_i, A_i] == 0, A_i], {EY == Sum[θ_s Y_s, {s, 1, S}], θ_S = 1 - Sum[θ_s, {s, 1, S-1}]}]
```

$$\left\{\left\{A_i \to \frac{(1 + Z B_{-i})(-EY + \alpha - Z A_{-i} + Z \alpha B_{-i}) + (Z \alpha + EY (Z + \beta) + Z (Z \alpha + EY \beta) B_{-i} + Z A_{-i} (Z + \beta + Z \beta B_{-i})) B_i}{(1 + Z B_{-i})(2 Z + \beta + Z \beta B_{-i})}\right\}\right\}$$

$$A_i = \frac{(1 + Z B_{-i})(-EY + \alpha - Z A_{-i} + Z \alpha B_{-i}) + (Z \alpha + EY (Z + \beta) + Z (Z \alpha + EY \beta) B_{-i} + Z A_{-i} (Z + \beta + Z \beta B_{-i})) B_i}{(1 + Z B_{-i})(2 Z + \beta + Z \beta B_{-i})}$$

$$\frac{(1 + Z B_{-i})(-EY + \alpha - Z A_{-i} + Z \alpha B_{-i}) + (Z \alpha + EY (Z + \beta) + Z (Z \alpha + EY \beta) B_{-i} + Z A_{-i} (Z + \beta + Z \beta B_{-i})) B_i}{(1 + Z B_{-i})(2 Z + \beta + Z \beta B_{-i})}$$

```
FullSimplify[D[u_i, B_i]]
```

0

The fact that derivative of u_i with respect to B_i is zero means $u_i(A_i^*(B_i), B_i, R_i)$ and $g_i(A_i^*(B_i), B_i)$ is independant of B_i. Therefore, $g_i(A_i^*(B_i), B_i)$ is a constant dependant on the cost and demand parameters.

```
FullSimplify[D[u_i, R_i], {EY == Sum[θ_s Y_s, {s, 1, S}], θ_S = 1 - Sum[θ_s, {s, 1, S-1}]}]
```

$$\frac{(-1 + (Z + \beta + \delta) R_i + Z R_{-i} (-1 + (\beta + \delta) R_i)) \left(EY^2 - \sum_{s=1}^{S} Y_s^2 \theta_s\right)}{(1 + Z (R_{-i} + R_i))^3}$$

The expression $(-1 + (Z + \beta + \delta) R_i + Z R_{-i} (-1 + (\beta + \delta) R_i))$ is a linear increasing function of R_i. Thus, it is negative bellow its zero and is positive after its zero. The denominator $(1 + Z (R_{-i} + R_i))^3$ is positive, and $\left(EY^2 - \sum_{s=1}^{S} \theta_s Y_s^2\right)$ is negative (because of Jensen's inequality). In sum, $\frac{du_i}{dR_i}$ is positive before its zero and is negative after this point. Thus, it is a quasi-concave function of R_i.

```
FullSimplify[Solve[D[u_i, R_i] == 0, R_i], {EY == Sum[θ_s Y_s, {s, 1, S}], θ_S = 1 - Sum[θ_s, {s, 1, S-1}]}]
```

$$\left\{\left\{R_i \to \frac{1 + Z R_{-i}}{Z + \beta + \delta + Z (\beta + \delta) R_{-i}}\right\}\right\}$$

R_i must be less than B_i, and u_i is a quasi-concave function of R_i. Therefore, the optimal R_i is

$$R_i = \mathrm{Min}\left[B_i, \frac{1 + Z R_{-i}}{Z + \beta + \delta + Z (\beta + \delta) R_{-i}}\right]$$

Finding a symmetric equilibrium

```
FullSimplify[Solve[{A_i ==
    ((1 + Z B_-i) (-EY + α - Z A_-i + Z α B_-i) + (Z α + EY (Z + β) + Z (Z α + EY β) B_-i + Z A_-i (Z + β + Z β B_-i))
      B_i) / ((1 + Z B_-i) (2 Z + β + Z β B_-i)), A_-i == (n - 1) A_i}, {A_i, A_-i}]]
```

$$\left\{\left\{A_i \to \frac{(1 + Z B_{-i})(-EY + \alpha + Z \alpha B_{-i}) + (Z \alpha + EY (Z + \beta) + Z (Z \alpha + EY \beta) B_{-i}) B_i}{(1 + Z B_{-i})(Z + n Z + \beta + Z \beta B_{-i}) - (-1 + n) Z (Z + \beta + Z \beta B_{-i}) B_i},\right.\right.$$

$$\left.\left. A_{-i} \to \frac{(-1 + n)(-(1 + Z B_{-i})(-EY + \alpha + Z \alpha B_{-i}) - (Z \alpha + EY (Z + \beta) + Z (Z \alpha + EY \beta) B_{-i}) B_i)}{-(1 + Z B_{-i})(Z + n Z + \beta + Z \beta B_{-i}) + (-1 + n) Z (Z + \beta + Z \beta B_{-i}) B_i}\right\}\right\}$$

```
n =.
FullSimplify[Solve[{R_i == (1 + Z R_-i)/(Z + β + δ + Z (β + δ) R_-i), R_-i == (n - 1) R_i}, {R_i, R_-i}]]
```

$$\left\{\left\{R_i \to -\frac{2}{(-2+n)\,Z-\beta-\delta+\sqrt{(-2+n)^2 Z^2+2\,n\,Z\,(\beta+\delta)+(\beta+\delta)^2}},\right.\right.$$

$$\left.R_{-i} \to -\frac{-(-2+n)\,Z+\beta+\delta+\sqrt{(-2+n)^2 Z^2+2\,n\,Z\,(\beta+\delta)+(\beta+\delta)^2}}{2\,Z\,(\beta+\delta)}\right\},$$

$$\left\{R_i \to \frac{2}{-(-2+n)\,Z+\beta+\delta+\sqrt{(-2+n)^2 Z^2+2\,n\,Z\,(\beta+\delta)+(\beta+\delta)^2}},\right.$$

$$\left.\left.R_{-i} \to \frac{(-2+n)\,Z-\beta-\delta+\sqrt{(-2+n)^2 Z^2+2\,n\,Z\,(\beta+\delta)+(\beta+\delta)^2}}{2\,Z\,(\beta+\delta)}\right\}\right\}$$

The expression $-\frac{2}{(-2+n)\,Z-\beta-\delta+\sqrt{(-2+n)^2 Z^2+2\,n\,Z\,(\beta+\delta)+(\beta+\delta)^2}}$ is negative. However, $\hat{R}_i = \frac{2}{-(-2+n)\,Z+\beta+\delta+\sqrt{(-2+n)^2 Z^2+2\,n\,Z\,(\beta+\delta)+(\beta+\delta)^2}}$ is positive and acceptable. As we show in the paper, the equilibrium R_i is $\mathrm{Min}\left[B_i, \frac{2}{-(-2+n)\,Z+\beta+\delta+\sqrt{(-2+n)^2 Z^2+2\,n\,Z\,(\beta+\delta)+(\beta+\delta)^2}}\right]$.

9 The LINGO model used to find the equilibrium of the two settlement mechanism with asymmetric firms

```
!The two settlement model with asymmetric generators and non-negativity constraints.;
MODEL:

DATA:
NumProblems= @OLE('TS1.xls', 'GENERATORS!K16');
ENDDATA

!The similar parameters to the parameters defined in the original paper have a similar
definition.
The rest of parameters are defined as comments.;
SETS:
GENERATORS :  b , alpha , beta , delta , q , a ,d, a_fixed,b_fixed,d_fixed, Opt,
optimizer, profit,lambda;
!
***a_fixed, b_fixed, d_fixed: The offered parametters of the generators in the last run.
***opt: If the current decision of the optimizing generator is similar (with a precision)
to its decision in the last run, it is 1, otherwise it is zero.
***optimizer: in each round it is one for the optimizing generator and zero for the
others.
***Lambda: The dual variable of the non-negativity constraint q_{i}>=0.
;
SCENARIOS : Y, theta, transCoef, p, C;
GEN_SCEN (GENERATORS, SCENARIOS): x, e, boundary ;
!
***e: The dual variable of the non-negativity consttraint q_i+x_{i,s}>=0.
***boundary: A binary variable to linearize the orthogonality constraint e_{i,s}(q_{i}+x_
{i,s})=0.
;
OPTIMIZERS  (GENERATORS);
!
The set of the optimizer generator in each step of the dynamic process.
;
FIXEDGENS(GENERATORS) | #NOT# @IN( OPTIMIZERS, &1) ;
!All non-optimizer generators;
ROWS /1..100/:alp,bet,del,op,a_f,b_f,d_f ,tet,coe,Y_f,Z_f,walpha ,wbeta ,wdelta
,woptimizer ,wa_fixed ,wb_fixed,wd_fixed ,wb ,wa ,wd ,wq ,wx1 ,wx2
,wprofit,wf,wp1,wp2,wwelfare,wrep,wst1,wst2,wtet,wcoe,wY_f,wZ_f;
!Degined for the purpose of collecting result of different runs of the model, and
outputting the results.;

ENDSETS

! Here is the data.
The data is read from an Excel file.
;
DATA:
GENERATORS, OPTIMIZERS= @OLE( 'TS1.xls', 'GENERATORS','OPTIMIZERS');
SCENARIOS = @OLE('TS1.xls','SCENARIOS');
theta, transCoef, Y = @OLE('TS1.xls','SCENSDATA');
Z, MyBigM = @OLE('TS1.xls' , 'Z' , 'MyBigM');
alp,bet,del,op,a_f,b_f,d_f = @OLE('TS1.xls','GENERATORS!D16:J116');
tet,coe,Y_f,Z_f = @OLE('TS1.xls','GENERATORS!N16:Q116');

precision =@OLE('TS1.xls','GENERATORS!R18');
!A tolerance that determines the smallest value that we consider as a change in strategy.
In other words, if the change in a firm's strategy is less than this, we count that as a
```

```
no change in the strategy.;

maxRep=@OLE('TS1.xls','GENERATORS!R19');
!If we do not find an equilibrium after "maxRep" steps, we stop searching for it.;

ENDDATA

SUBMODEL TS1:
!This is the optimization model solved by a firm to maximize profit, assuming that the
strategy set of all other firms are fixed. ;
@FOR(GENERATORS: @FREE(a));
@FOR(SCENARIOS: @FREE(p));
@FOR(GEN_SCEN: @FREE(x));
@FREE(f);

[obj] MAX = @sum (GENERATORS(i): optimizer(i)* (  f * q(i)+ @sum (SCENARIOS(s): (theta(s)*
(p(s)* x(i,s)-(alpha(i) * (q(i)+x(i,s))+ beta(i)/2 * (q(i)+x(i,s))^2 + delta(i)/2 *x(i,s)^
2)))  ) ));
! The objective;

!The constraints include constraints of a generator on his offered supply function and KKT
consitions of the ISO's optimization problem;
@FOR (GENERATORS(i):
      -f+a(i)+b(i)*q(i)-lambda(i)=0;
      q(i)*lambda(i)=0;
);

@FOR(GEN_SCEN(i,s):
       a(i)-p(s)-e(i,s)+b(i)*(q(i)+x(i,s))=0;
      q(i) + x(i,s) >= 0;
      [Const_ebin]      e(i,s)  <= boundary(i,s)*MyBigM;
      [Const_qxbin]     q(i)+x(i,s)  <= (1-boundary(i,s))*MyBigM;
      @BIN(boundary(i,s));
);


@FOR(SCENARIOS(S):
[Const_p_demand]  theta(s) * (p(s)+ Z* C(s)-Y(s))   = 0;
[Const_C]   theta(s)*(-cQ+C(s)-@sum (GENERATORS(i): x(i,s))) = 0;
);

!Non-optimizing generators should offer their previous offered parameters;
@FOR (GENERATORS(k)|optimizer(k) #EQ# 0 :
      a(k)=a_fixed(k);
      b(k)=b_fixed(k);
);

-@sum(SCENARIOS(s):theta(s)*Y(s))+f+cQ*Z=0;
cQ - @sum(GENERATORS(h): q(h))= 0;

ENDSUBMODEL

!Calculations and procedure of the dynamic process to find an equilibrium for each of the
market settings.;
CALC:
@for(ROWS(k):

walpha(k)=0;
wbeta(k)=0;
wdelta(k)=0;
woptimizer(k)=0;
wa_fixed(k)=0;
wb_fixed(k)=0;
wd_fixed(k)=0;
wb(k)=0;
wa(k)=0;
```

```
wd(k)=0;
wq(k)=0;
wx1(k)=0;
wx2(k)=0;
wprofit(k)=0;
wf(k)=0;
wp1(k)=0;
wp2(k)=0;
wwelfare(k)=0;
wrep(k)=0;
wst1(k)=0;
wst2(k)=0;
wtet(k)=0;
wcoe(k)=0;
wY_f(k)=0;
wZ_f(k)=0;
);

!Reading different market settings (i.e. case studies or examples).;
ind=@OLE('TS1.xls','GENERATORS!L16');
@WHILE (ind #LE# NumProblems:
eq=0;
rep=0;
alp1=alp(2*(ind-1)+1);
alp2=alp(2*(ind-1)+2);
bet1=bet(2*(ind-1)+1);
bet2=bet(2*(ind-1)+2);
del1=del(2*(ind-1)+1);
del2=del(2*(ind-1)+2);
op1=op(2*(ind-1)+1);
op2=op(2*(ind-1)+2);
a_f1=a_f(2*(ind-1)+1);
a_f2=a_f(2*(ind-1)+2);
b_f1=b_f(2*(ind-1)+1);
b_f2=b_f(2*(ind-1)+2);
d_f1=d_f(2*(ind-1)+1);
d_f2=d_f(2*(ind-1)+2);
tet1=tet(2*(ind-1)+1);
tet2=tet(2*(ind-1)+2);
coe1=coe(2*(ind-1)+1);
coe2=coe(2*(ind-1)+2);
Y_f1=Y_f(2*(ind-1)+1);
Y_f2=Y_f(2*(ind-1)+2);
Z_f1=Z_f(2*(ind-1)+1);

@OLE('TS1.xls','GENERATORS!D2:j2')=alp1,bet1,del1,op1,a_f1,b_f1,d_f1;
@OLE('TS1.xls','GENERATORS!D3:j3')=alp2,bet2,del2,op2,a_f2,b_f2,d_f2;

@OLE('TS1.xls','SCENARIOS!C2:E2')=tet1,coe1,Y_f1;
@OLE('TS1.xls','SCENARIOS!C3:E3')=tet2,coe2,Y_f2;

@OLE('TS1.xls','OtherParams!B2')=Z_f1;


@for( GENERATORS(i):
      Opt(i)=0;
);
! st1 and st2 records the status of the optimization problems i.e. whether it is found a
global optimal solution or a local optima. These are importnt to ensure that we actually
find a true equilibrium.;
st1=1000;
st2=1000;
@WHILE (eq #LE# 1 #AND# rep#LE#maxRep:
      st1=st2;
      alpha, beta, delta, optimizer, a_fixed, b_fixed,d_fixed = @OLE( 'TS1.xls',
'GENSDATA');
      theta, transCoef, Y = @OLE('TS1.xls','SCENSDATA');
```

```
    Z = @OLE('TS1.xls' , 'Z');
    @SOLVE( TS1);

    @for(GENERATORS(i)| optimizer(i) #EQ# 1 :
        @ifc( a(i) #GE# a_fixed(i)-precision  #AND#  a(i) #LE# a_fixed(i)+precision
#AND# b(i) #GE# b_fixed(i)-precision  #AND#  b(i) #LE# b_fixed(i)+precision:
            Opt(i)=1;
        @else
            Opt(i)=0;
         );

        a_fixed(i) = a(i);
        b_fixed(i) = b(i);
    );
    @for(GENERATORS(i):
        @ifc( optimizer(i) #EQ# 1:
            optimizer(i)=0;
        @else
            optimizer(i)=1;
         );
    );
    st2=@STATUS();
    eq = @sum(GENERATORS(i): Opt(i));
    @OLE( 'TS1.xls', 'GENSDATA') = alpha, beta, delta, optimizer, a_fixed, b_fixed,
d_fixed;
    rep=rep+1;
    @for(GENERATORS(i):
        profit(i) =  f * q(i)+ @sum (SCENARIOS(s): (theta(s)*(p(s)* x(i,s)-(alpha(i) *
(q(i)+x(i,s))+ beta(i)/2 * (q(i)+x(i,s))^2 + delta(i)/2 *x(i,s)^2)))  ) ;
    );

    !Intermediate output;
    welfare = @sum(SCENARIOS(s): theta(s)*(Y(s)*C(s)-Z/2*C(s)^2-@sum(GENERATORS(i):
alpha(i)*(q(i)+x(i,s))+beta(i)/2*(q(i)+x(i,s))^2+delta(i)/2*x(i,s)^2)));
    @OLE( 'TS1.xls', 'GENERATORS!L2:N3') = a, d, q;
    @OLE( 'TS1.xls', 'GENERATORS!O2:O3') = @writefor(GEN_SCEN(i,s)|s #EQ# 1: x(i,s));
    @OLE( 'TS1.xls', 'GENERATORS!P2:P3') = @writefor(GEN_SCEN(i,s)|s #EQ# 2: x(i,s));
    @OLE( 'TS1.xls', 'GENERATORS!Q2:Q3') =profit;
    @OLE( 'TS1.xls', 'GENERATORS!R2:R2') = f;
    @OLE( 'TS1.xls', 'GENERATORS!S2:T2') = p;
    @OLE( 'TS1.xls', 'GENERATORS!U2:U2') = welfare;
    @OLE( 'TS1.xls', 'GENERATORS!V2:V2') = rep;
    @OLE( 'TS1.xls', 'GENERATORS!W2:W2') = st1;
    @OLE( 'TS1.xls', 'GENERATORS!X2:X2') = st2;
    @OLE( 'TS1.xls', 'GENERATORS!Y2:Y2') = @write('WNN');
);

!Final output;
welfare = @sum(SCENARIOS(s): theta(s)*(Y(s)*C(s)-Z/2*C(s)^2-@sum(GENERATORS(i): alpha(i)*
(q(i)+x(i,s))+beta(i)/2*(q(i)+x(i,s))^2+delta(i)/2*x(i,s)^2)));
@OLE( 'TS1.xls', 'GENERATORS!L2:N3') = a, b, q;
@OLE( 'TS1.xls', 'GENERATORS!O2:O3') = @writefor(GEN_SCEN(i,s)|s #EQ# 1: x(i,s));
@OLE( 'TS1.xls', 'GENERATORS!P2:P3') = @writefor(GEN_SCEN(i,s)|s #EQ# 2: x(i,s));
@OLE( 'TS1.xls', 'GENERATORS!Q2:Q3') = profit;
@OLE( 'TS1.xls', 'GENERATORS!R2:R2') = f;
@OLE( 'TS1.xls', 'GENERATORS!S2:T2') = p;
@OLE( 'TS1.xls', 'GENERATORS!U2:U2') = welfare;
@OLE( 'TS1.xls', 'GENERATORS!V2:V2') = rep;
@OLE( 'TS1.xls', 'GENERATORS!W2:W2') = st1;
@OLE( 'TS1.xls', 'GENERATORS!X2:X2') = st2;
@OLE( 'TS1.xls', 'GENERATORS!Y2:Y2') = @write('WNN');
i=1;
    walpha(2*(ind-1)+i)=alpha(i);
    wbeta(2*(ind-1)+i)=beta(i);
    wdelta(2*(ind-1)+i)=delta(i);
    woptimizer(2*(ind-1)+i)=optimizer(i);
    wa_fixed(2*(ind-1)+i)=a_f(2*(ind-1)+i);
```

```
        wb_fixed(2*(ind-1)+i)=b_f(2*(ind-1)+i);
        wd_fixed(2*(ind-1)+i)=d_f(2*(ind-1)+i);
        wa(2*(ind-1)+i)=a(i);
        wb(2*(ind-1)+i)=b(i);
        wd(2*(ind-1)+i)=d(i);
        wq(2*(ind-1)+i)=q(i);
        wx1(2*(ind-1)+i)=x(i,1);
        wx2(2*(ind-1)+i)=x(i,2);
        wprofit(2*(ind-1)+i)=profit(i);
        wf(2*(ind-1)+i)=f;
        wp1(2*(ind-1)+i)=p(1);
        wp2(2*(ind-1)+i)=p(2);
        wwelfare(2*(ind-1)+i)=welfare;
        wrep(2*(ind-1)+i)=rep;
        wst1(2*(ind-1)+i)=st1;
        wst2(2*(ind-1)+i)=st2;
        wtet(2*(ind-1)+i)=theta(i);
        wcoe(2*(ind-1)+i)=transCoef(i);
        wY_f(2*(ind-1)+i)=Y(i);
        wZ_f(2*(ind-1)+i)=Z;

i=2;
        walpha(2*(ind-1)+i)=alpha(i);
        wbeta(2*(ind-1)+i)=beta(i);
        wdelta(2*(ind-1)+i)=delta(i);
        woptimizer(2*(ind-1)+i)=optimizer(i);
        wa_fixed(2*(ind-1)+i)=a_f(2*(ind-1)+i);
        wb_fixed(2*(ind-1)+i)=b_f(2*(ind-1)+i);
        wd_fixed(2*(ind-1)+i)=d_f(2*(ind-1)+i);
        wa(2*(ind-1)+i)=a(i);
        wb(2*(ind-1)+i)=b(i);
        wd(2*(ind-1)+i)=d(i);
        wq(2*(ind-1)+i)=q(i);
        wx1(2*(ind-1)+i)=x(i,1);
        wx2(2*(ind-1)+i)=x(i,2);
        wprofit(2*(ind-1)+i)=profit(i);
        wf(2*(ind-1)+i)=f;
        wp1(2*(ind-1)+i)=p(1);
        wp2(2*(ind-1)+i)=p(2);
        wwelfare(2*(ind-1)+i)=welfare;
        wrep(2*(ind-1)+i)=rep;
        wst1(2*(ind-1)+i)=st1;
        wst2(2*(ind-1)+i)=st2;
        wtet(2*(ind-1)+i)=theta(i);
        wcoe(2*(ind-1)+i)=transCoef(i);
        wY_f(2*(ind-1)+i)=Y(i);
        wZ_f(2*(ind-1)+i)=Z;

ind=ind+1;
@OLE( 'TS1.xls', 'OUT_WNN!B2:E101') = wtet,wcoe,wY_f,wZ_f;
@OLE( 'TS1.xls', 'OUT_WNN!G2:AA101') =  walpha ,wbeta ,wdelta ,woptimizer ,wa_fixed
,wb_fixed,wd_fixed ,wa ,wb ,wd ,wq ,wx1 ,wx2 ,wprofit,wf,wp1,wp2,wwelfare,wrep,wst1,wst2;
);



ENDCALC
@WARN('LINGO Finished',1#GE#0);
END
```

10 The LINGO model used to find the equilibrium of the stochastic settlement mechanism with asymmetric firms

```
!The stochastic settlement model with asymmetric generators and non-negativity
constraints.;
MODEL:
DATA:
NumProblems= @OLE('SFSP.xls', 'GENERATORS!K16');
ENDDATA

!The similar parameters to the parameters defined in the original paper have a similar
definition.
The rest of parameters are defined as comments.;
SETS:

GENERATORS :  b , alpha , beta , delta , q , a , d, a_fixed, d_fixed, Opt, optimizer,
profit;
!
***a_fixed, b_fixed, d_fixed: The offered paramettersof the generators in the last run.
***opt: If the current decision of the optimizing generator is similar (with a precision)
to its decision in the last run, it is 1, otherwise it is zero.
***optimizer: in each round it is one for the optimizing generator and zero for the
others.
***
;
SCENARIOS : Y, theta, transCoef, p, C;
GEN_SCEN (GENERATORS, SCENARIOS): x, e, boundary ;
!
***e: The dual variable of the non-negativity consttraint q_i+x_{i,s}>=0.
***boundary: A binary variable to linearize the orthogonality constraint e_{i,s}(q_{i}+x_
{i,s})=0.
;
OPTIMIZERS  (GENERATORS);
!
The set of the optimizer generator in each step of the dynamic process.
;
FIXEDGENS(GENERATORS) | #NOT# @IN( OPTIMIZERS, &1) ;
!All non-optimizer generators;
ROWS /1..100/:alp,bet,del,op,a_f,b_f,d_f ,tet,coe,Y_f,Z_f,walpha ,wbeta ,wdelta
,woptimizer ,wa_fixed ,wb_fixed,wd_fixed ,wb ,wa ,wd ,wq ,wx1 ,wx2
,wprofit,wf,wp1,wp2,wwelfare,wrep,wst1,wst2,wtet,wcoe,wY_f,wZ_f;
!Degined for the purpose of collecting result of different runs of the model, and
outputting the results.;

ENDSETS

! Here is the data.
The data is read from an Excel file.
;
DATA:
GENERATORS, OPTIMIZERS= @OLE( 'SFSP.xls', 'GENERATORS','OPTIMIZERS');
SCENARIOS = @OLE('SFSP.xls','SCENARIOS');
theta, transCoef, Y = @OLE('SFSP.xls','SCENSDATA');
Z, MyBigM = @OLE('SFSP.xls' , 'Z' , 'MyBigM');
alp,bet,del,op,a_f,b_f,d_f = @OLE('SFSP.xls','GENERATORS!D16:J116');
tet,coe,Y_f,Z_f = @OLE('SFSP.xls','GENERATORS!N16:Q116');
precision =@OLE('SFSP.xls','GENERATORS!R18');
!A tolerance that determines the smallest value that we consider as a change in strategy.
In other words, if the change in a firm's strategy is less than this, we count that as a
```

```
no change in the strategy.;

maxRep=@OLE('SFSP.xls','GENERATORS!R19');
!If we do not find an equilibrium after "maxRep" steps, we stop searching for it.;

ENDDATA

SUBMODEL SFSP:
!This is the optimization model solved by a firm to maximize profit, assuming that the
strategy set of all other firms are fixed. ;
@FOR(GENERATORS: @FREE(a));
@FOR(SCENARIOS: @FREE(p));
@FOR(GEN_SCEN: @FREE(x));
@FREE(f);

[obj] MAX = @sum (GENERATORS(i): optimizer(i)* (  @sum (SCENARIOS(s): (theta(s)*p(s))) * q
(i)+ @sum (SCENARIOS(s): (theta(s)*(p(s)* transCoef(s)*x(i,1)-(alpha(i) * (q(i)+x(i,s))+
beta(i)/2 * (q(i)+transCoef(s)*x(i,1))^2 + delta(i)/2 *transCoef(s)^2*x(i,1)^2)))  ) ));
! The objective;

!The constraints include constraints of a generator on his offered supply function and KKT
consitions of the ISO's optimization problem;
@FOR (GENERATORS(i):
[Const_f]   -f + @sum ( SCENARIOS(s): (-e(i,s)+b(i)*theta(s)*x(i,s))) + a(i) + b(i)*q(i) =
0;
      x(i,2)=transCoef(2)*x(i,1);
);

@FOR(GEN_SCEN(i,s):
[Const_p]   -e(i,s) +theta(s)*(-p(s)+a(i)+b(i)*q(i)+(b(i)+d(i))*transCoef(s)*x(i,1))  = 0;
      q(i) + x(i,s) >= 0;
[Const_ebin]      e(i,s)  <= boundary(i,s)*MyBigM;
[Const_qxbin]     q(i)+x(i,s) <= (1-boundary(i,s))*MyBigM;
      @BIN(boundary(i,s));
);


@FOR(SCENARIOS(S):
[Const_p_demand]  theta(s) * (p(s)+ Z* C(s)-Y(s))  = 0;
[Const_C]   theta(s)*(-cQ+C(s)-@sum (GENERATORS(i): x(i,s))) = 0;
);

!Non-optimizing generators should offer their previous offered parameters;
@FOR (GENERATORS(k)|optimizer(k) #EQ# 0 :
      a(k)=a_fixed(k);
      d(k)=d_fixed(k);
);

f - @sum (SCENARIOS(s): (theta(s)*p(s)))  = 0;
cQ - @sum (GENERATORS(h): q(h))  = 0;

ENDSUBMODEL

!Calculations and procedure of the dynamic process to find an equilibrium for each of the
market settings.;
CALC:
@for(ROWS(k):

walpha(k)=0;
wbeta(k)=0;
wdelta(k)=0;
woptimizer(k)=0;
wa_fixed(k)=0;
wb_fixed(k)=0;
wd_fixed(k)=0;
wb(k)=0;
wa(k)=0;
```

```
wd(k)=0;
wq(k)=0;
wx1(k)=0;
wx2(k)=0;
wprofit(k)=0;
wf(k)=0;
wp1(k)=0;
wp2(k)=0;
wwelfare(k)=0;
wrep(k)=0;
wst1(k)=0;
wst2(k)=0;
wtet(k)=0;
wcoe(k)=0;
wY_f(k)=0;
wZ_f(k)=0;
);

!Reading different market settings (i.e. case studies or examples).;
ind=@OLE('SFSP.xls','GENERATORS!L16');
@WHILE (ind #LE# NumProblems:
eq=0;
rep=0;
alp1=alp(2*(ind-1)+1);
alp2=alp(2*(ind-1)+2);
bet1=bet(2*(ind-1)+1);
bet2=bet(2*(ind-1)+2);
del1=del(2*(ind-1)+1);
del2=del(2*(ind-1)+2);
op1=op(2*(ind-1)+1);
op2=op(2*(ind-1)+2);
a_f1=a_f(2*(ind-1)+1);
a_f2=a_f(2*(ind-1)+2);
b_f1=b_f(2*(ind-1)+1);
b_f2=b_f(2*(ind-1)+2);
d_f1=d_f(2*(ind-1)+1);
d_f2=d_f(2*(ind-1)+2);
tet1=tet(2*(ind-1)+1);
tet2=tet(2*(ind-1)+2);
coe1=coe(2*(ind-1)+1);
coe2=coe(2*(ind-1)+2);
Y_f1=Y_f(2*(ind-1)+1);
Y_f2=Y_f(2*(ind-1)+2);
Z_f1=Z_f(2*(ind-1)+1);

@OLE('SFSP.xls','GENERATORS!D2:j2')=alp1,bet1,del1,op1,a_f1,b_f1,d_f1;
@OLE('SFSP.xls','GENERATORS!D3:j3')=alp2,bet2,del2,op2,a_f2,b_f2,d_f2;

@OLE('SFSP.xls','SCENARIOS!C2:E2')=tet1,coe1,Y_f1;
@OLE('SFSP.xls','SCENARIOS!C3:E3')=tet2,coe2,Y_f2;

@OLE('SFSP.xls','OtherParams!B2')=Z_f1;

@for( GENERATORS(i):
      Opt(i)=0;
);

! st1 and st2 records the status of the optimization problems i.e. whether it is found a
global optimal solution or a local optima. These are importnt to ensure that we actually
find a true equilibrium.;
st1=1000;
st2=1000;

@WHILE (eq #LE# 1 #AND# rep#LE#maxRep:
      st1=st2;
      alpha, beta, delta, optimizer, a_fixed, b, d_fixed = @OLE( 'SFSP.xls', 'GENSDATA');
```

```
        theta, transCoef, Y = @OLE('SFSP.xls','SCENSDATA');
        Z = @OLE('SFSP.xls' , 'Z');
        @SOLVE( SFSP);

        @for(GENERATORS(i)| optimizer(i) #EQ# 1 :
                @ifc( a(i) #GE# a_fixed(i)-precision  #AND#  a(i) #LE# a_fixed(i)+precision
#AND# d(i) #GE# d_fixed(i)-precision  #AND#  d(i) #LE# d_fixed(i)+precision:
                        Opt(i)=1;
                @else
                        Opt(i)=0;
                 );

                a_fixed(i) = a(i);
                d_fixed(i) = d(i);
        );
        @for(GENERATORS(i):
                @ifc( optimizer(i) #EQ# 1:
                        optimizer(i)=0;
                @else
                        optimizer(i)=1;
                 );

        );
        st2=@STATUS();
        eq = @sum(GENERATORS(i): Opt(i));
        @OLE( 'SFSP.xls', 'GENSDATA') = alpha, beta, delta, optimizer, a_fixed, b,d_fixed ;

        rep=rep+1;
        @for(GENERATORS(i):
                profit(i) =  f * q(i)+ @sum (SCENARIOS(s): (theta(s)*(p(s)* x(i,s)-(alpha(i) *
(q(i)+x(i,s))+ beta(i)/2 * (q(i)+x(i,s))^2 + delta(i)/2 *x(i,s)^2)))  ) ;
        );
        !Intermediate output;
        welfare = @sum(SCENARIOS(s): theta(s)*(Y(s)*C(s)-Z/2*C(s)^2-@sum(GENERATORS(i):
alpha(i)*(q(i)+x(i,s))+beta(i)/2*(q(i)+x(i,s))^2+delta(i)/2*x(i,s)^2)));
        @OLE( 'SFSP.xls', 'GENERATORS!L2:N3') = a, d, q;
        @OLE( 'SFSP.xls', 'GENERATORS!O2:O3') = @writefor(GEN_SCEN(i,s)|s #EQ# 1: x(i,s));
        @OLE( 'SFSP.xls', 'GENERATORS!P2:P3') = @writefor(GEN_SCEN(i,s)|s #EQ# 2: x(i,s));
        @OLE( 'SFSP.xls', 'GENERATORS!Q2:Q3') =profit;
        @OLE( 'SFSP.xls', 'GENERATORS!R2:R2') = f;
        @OLE( 'SFSP.xls', 'GENERATORS!S2:T2') = p;
        @OLE( 'SFSP.xls', 'GENERATORS!U2:U2') = welfare;
        @OLE( 'SFSP.xls', 'GENERATORS!V2:V2') = rep;
        @OLE( 'SFSP.xls', 'GENERATORS!W2:W2') = st1;
        @OLE( 'SFSP.xls', 'GENERATORS!X2:X2') = st2;
        @OLE( 'SFSP.xls', 'GENERATORS!Y2:Y2') = @write('WNN');
);

!Final output;
welfare = @sum(SCENARIOS(s): theta(s)*(Y(s)*C(s)-Z/2*C(s)^2-@sum(GENERATORS(i): alpha(i)*
(q(i)+x(i,s))+beta(i)/2*(q(i)+x(i,s))^2+delta(i)/2*x(i,s)^2)));
@OLE( 'SFSP.xls', 'GENERATORS!L2:N3') = a, d, q;
@OLE( 'SFSP.xls', 'GENERATORS!O2:O3') = @writefor(GEN_SCEN(i,s)|s #EQ# 1: x(i,s));
@OLE( 'SFSP.xls', 'GENERATORS!P2:P3') = @writefor(GEN_SCEN(i,s)|s #EQ# 2: x(i,s));
@OLE( 'SFSP.xls', 'GENERATORS!Q2:Q3') =profit;
@OLE( 'SFSP.xls', 'GENERATORS!R2:R2') = f;
@OLE( 'SFSP.xls', 'GENERATORS!S2:T2') = p;
@OLE( 'SFSP.xls', 'GENERATORS!U2:U2') = welfare;
@OLE( 'SFSP.xls', 'GENERATORS!V2:V2') = rep;
@OLE( 'SFSP.xls', 'GENERATORS!W2:W2') = st1;
@OLE( 'SFSP.xls', 'GENERATORS!X2:X2') = st2;
@OLE( 'SFSP.xls', 'GENERATORS!Y2:Y2') = @write('WNN');
i=1;
        walpha(2*(ind-1)+i)=alpha(i);
        wbeta(2*(ind-1)+i)=beta(i);
        wdelta(2*(ind-1)+i)=delta(i);
        woptimizer(2*(ind-1)+i)=optimizer(i);
```

```
        wa_fixed(2*(ind-1)+i)=a_f(2*(ind-1)+i);
        wb_fixed(2*(ind-1)+i)=b_f(2*(ind-1)+i);
        wd_fixed(2*(ind-1)+i)=d_f(2*(ind-1)+i);
        wa(2*(ind-1)+i)=a(i);
        wb(2*(ind-1)+i)=b(i);
        wd(2*(ind-1)+i)=d(i);
        wq(2*(ind-1)+i)=q(i);
        wx1(2*(ind-1)+i)=x(i,1);
        wx2(2*(ind-1)+i)=x(i,2);
        wprofit(2*(ind-1)+i)=profit(i);
        wf(2*(ind-1)+i)=f;
        wp1(2*(ind-1)+i)=p(1);
        wp2(2*(ind-1)+i)=p(2);
        wwelfare(2*(ind-1)+i)=welfare;
        wrep(2*(ind-1)+i)=rep;
        wst1(2*(ind-1)+i)=st1;
        wst2(2*(ind-1)+i)=st2;
        wtet(2*(ind-1)+i)=theta(i);
        wcoe(2*(ind-1)+i)=transCoef(i);
        wY_f(2*(ind-1)+i)=Y(i);
        wZ_f(2*(ind-1)+i)=Z;

i=2;
        walpha(2*(ind-1)+i)=alpha(i);
        wbeta(2*(ind-1)+i)=beta(i);
        wdelta(2*(ind-1)+i)=delta(i);
        woptimizer(2*(ind-1)+i)=optimizer(i);
        wa_fixed(2*(ind-1)+i)=a_f(2*(ind-1)+i);
        wb_fixed(2*(ind-1)+i)=b_f(2*(ind-1)+i);
        wd_fixed(2*(ind-1)+i)=d_f(2*(ind-1)+i);
        wa(2*(ind-1)+i)=a(i);
        wb(2*(ind-1)+i)=b(i);
        wd(2*(ind-1)+i)=d(i);
        wq(2*(ind-1)+i)=q(i);
        wx1(2*(ind-1)+i)=x(i,1);
        wx2(2*(ind-1)+i)=x(i,2);
        wprofit(2*(ind-1)+i)=profit(i);
        wf(2*(ind-1)+i)=f;
        wp1(2*(ind-1)+i)=p(1);
        wp2(2*(ind-1)+i)=p(2);
        wwelfare(2*(ind-1)+i)=welfare;
        wrep(2*(ind-1)+i)=rep;
        wst1(2*(ind-1)+i)=st1;
        wst2(2*(ind-1)+i)=st2;
        wtet(2*(ind-1)+i)=theta(i);
        wcoe(2*(ind-1)+i)=transCoef(i);
        wY_f(2*(ind-1)+i)=Y(i);
        wZ_f(2*(ind-1)+i)=Z;

ind=ind+1;
@OLE( 'SFSP.xls', 'OUT_WNN!B2:E101') = wtet,wcoe,wY_f,wZ_f;
@OLE( 'SFSP.xls', 'OUT_WNN!G2:AA101') =  walpha ,wbeta ,wdelta ,woptimizer ,wa_fixed
,wb_fixed,wd_fixed ,wa ,wb ,wd ,wq ,wx1 ,wx2 ,wprofit,wf,wp1,wp2,wwelfare,wrep,wst1,wst2;
);

ENDCALC
@WARN('LINGO Finished',1#GE#0);
END
```